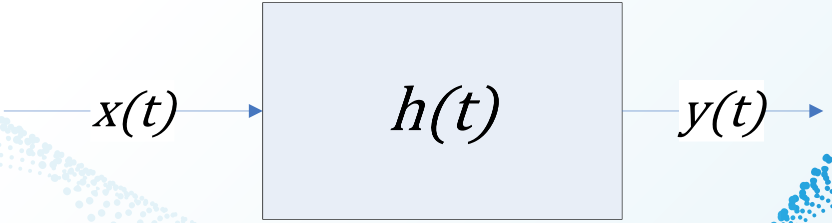
Transformada de Fourier de Tiempo Continuo

Respuesta en frecuencia

Ejemplo

- Considerando al Sistema continuo LTI descripto por

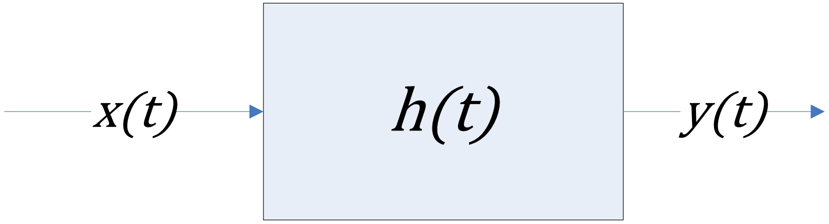
$$\frac{dy(t)}{dt} + y(t) = x(t)$$

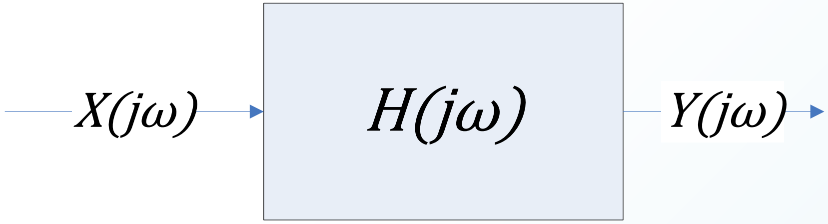


a) Hallar $H(j\omega)$

b) Hallar $h(t)$

c) Hallar $y(t)$, sabiendo que $x(t) = e^{-t}u(t)$.





$$y(t) = h(t) * x(t)$$

$$Y(j\omega) = H(j\omega) \cdot X(j\omega)$$

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

a) Hallar $H(j\omega)$

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Transformando:

$$j\omega Y(j\omega) + 2Y(j\omega) = X(j\omega)$$

$$(j\omega + 2)Y(j\omega) = X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{j\omega + 2}$$

b) Hallar $h(t)$:

Anti transformando: $$h(t) = e^{-2t}u(t)$$

c) Hallar $y(t)$, sabiendo que $x(t) = e^{-t}u(t)$

$$Y(j\omega) = H(j\omega) \cdot X(j\omega) \qquad H(j\omega) = \frac{1}{j\omega + 2} \qquad X(j\omega) = \frac{1}{j\omega + 1}$$

$$Y(j\omega) = \frac{1}{j\omega + 2} \cdot \frac{1}{j\omega + 1}$$

Fracciones Parciales:

$$Y(j\omega) = \frac{1}{j\omega + 2} \cdot \frac{1}{j\omega + 1} = \frac{A}{j\omega + 2} + \frac{B}{j\omega + 1} =$$

$$= \frac{A(j\omega + 1) + B(j\omega + 2)}{(j\omega + 2)(j\omega + 1)}$$

$$j\omega(A + B) = 0$$

$$A(j\omega + 1) + B(j\omega + 2) = 1$$

$$A + 2B = 1$$

$$\begin{cases} A + B = 0 \\ A + 2B = 1 \end{cases}$$

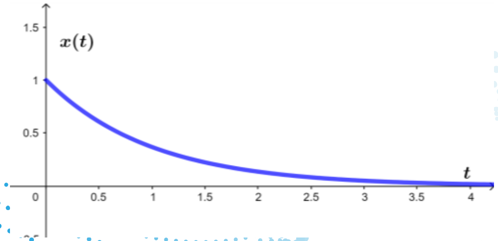
$$A = -B = -1$$

$$B = 1$$

Fracciones Parciales:

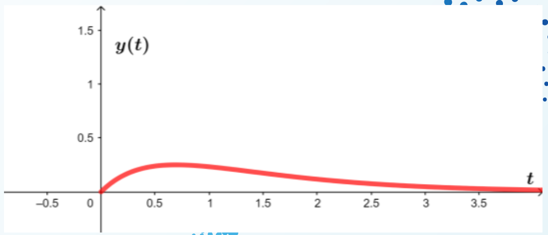
$$Y(j\omega) = \frac{1}{j\omega + 2} \cdot \frac{1}{j\omega + 1} = \frac{-1}{j\omega + 2} + \frac{1}{j\omega + 1} =$$

$$y(t) = (e^{-t} - e^{-2t})u(t)$$



$x(t)$ → $h(t)$ → $y(t)$

$x(t) = e^{-t}u(t).$



$y(t) = (e^{-t} - e^{-2t})u(t)$