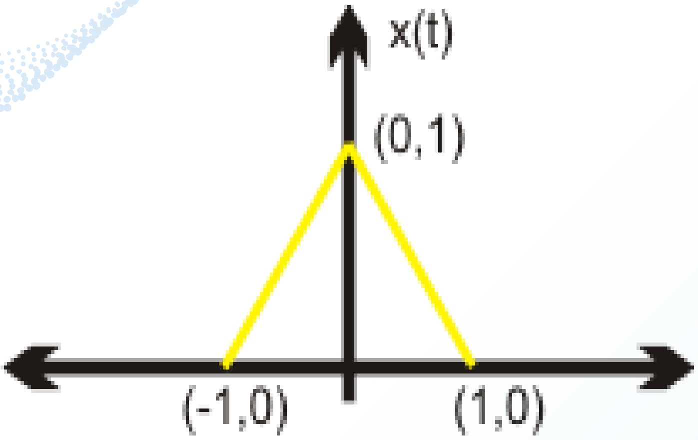
Convolución de Tiempo Continuo

Ejemplo

Utilizando las siguientes señales, se pide calcular la siguiente convolución de tiempo continuo: $x(t) * x(t)$



$$x(t) = \begin{cases} t + 1 & -1 < t \leq 0 \\ -t + 1 & 0 < t \leq 1 \\ 0 & \forall\, t \end{cases}$$

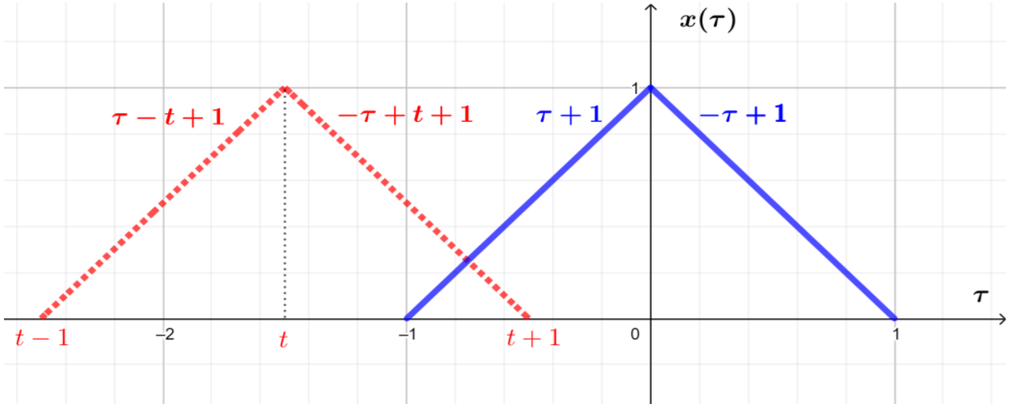
Desarrollo:

- Primeramente, debemos recordar que la misma está definida como:

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) d\tau$$

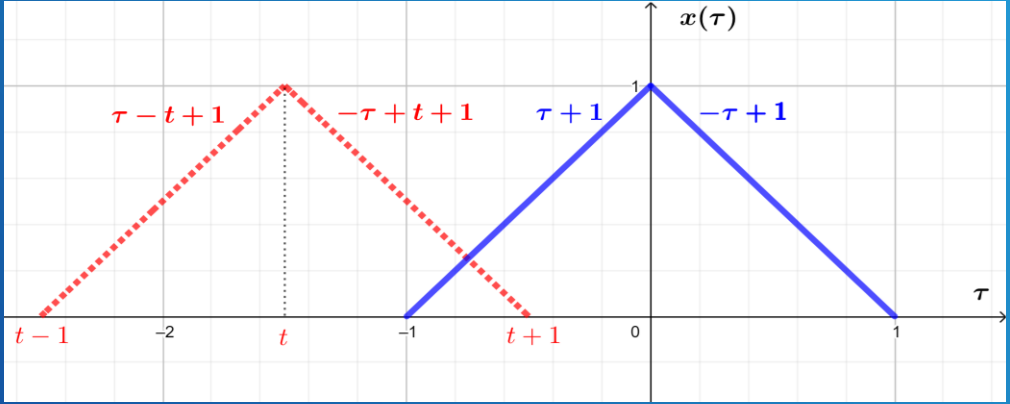
- $x(t) * x(t) = \int_{-\infty}^{\infty} x(\tau) \cdot x(t-\tau) d\tau$

Convolución Continua



$$x(\tau) = \begin{cases} \tau + 1 & -1 < \tau \leq 0 \\ -\tau + 1 & 0 < \tau \leq 1 \\ 0 & \forall \tau \end{cases}$$

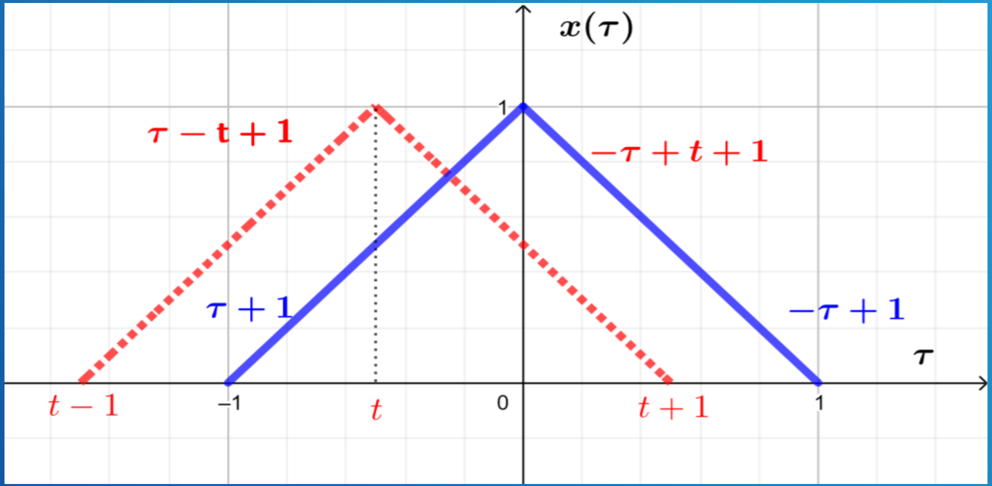
$$x(t - \tau) = \begin{cases} \tau - t + 1 & t - 1 < \tau \leq t \\ -\tau + t + 1 & t < \tau \leq t + 1 \\ 0 & \forall \tau \end{cases}$$



$$\frac{t^3 + 6t^2 + 12t + 8}{6} \quad -2 < t \leq -1$$

$$\int_{-1}^{t+1} (-\tau + t + 1)(\tau + 1)d\tau = \int_{-1}^{t+1} (-\tau^2 + t\tau + t + 1)d\tau$$

$$= -\frac{\tau^3}{3}\Big|_{-1}^{t+1} + t\frac{\tau^2}{2}\Big|_{-1}^{t+1} + (t+1)\,\tau\Big|_{-1}^{t+1}$$

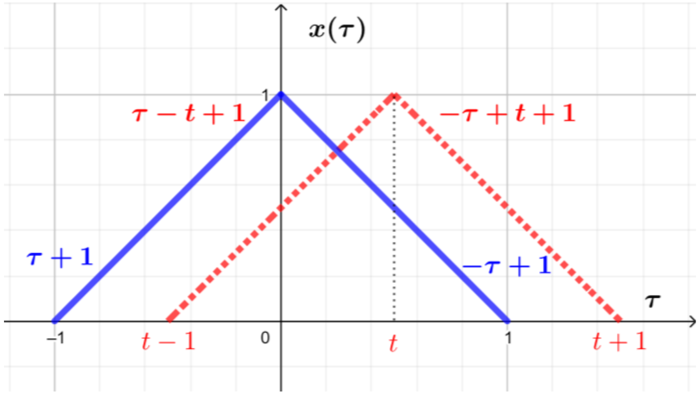


$$-\frac{3t^3+6t^2-4}{6}, -1 < t \leq 0$$

$$\int_{-1}^{t} (\tau - t + 1)(\tau + 1)d\tau + \int_{t}^{0} (-\tau + t + 1)(\tau + 1)d\tau + \int_{0}^{t+1} (-\tau + t + 1)(-\tau + 1)d\tau$$

$$\int_{-1}^{t} (\tau^2 + (2-t)\tau - t + 1)d\tau + \int_{t}^{0} (-\tau^2 + t\tau + t + 1)d\tau + \int_{0}^{t+1} (\tau^2 - (2+t)\tau + t + 1)d\tau$$

$$= -\frac{\tau^3}{3}\Big|_{-1}^{t} + (2-t)\frac{\tau^2}{2}\Big|_{-1}^{t} + (-t+1)\,\tau\Big|_{-1}^{t} - \frac{\tau^3}{3}\Big|_{t}^{0} + t\frac{\tau^2}{2}\Big|_{t}^{0} + (t+1)\,\tau\Big|_{t}^{0} + \frac{\tau^3}{3}\Big|_{0}^{t+1} - (2+t)\frac{\tau^2}{2}\Big|_{0}^{t+1} + (t+1)\,\tau\Big|_{0}^{t+1}$$



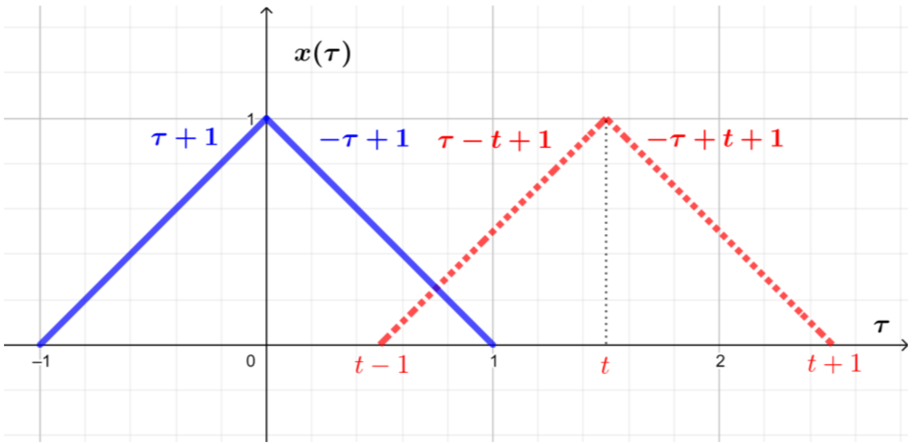
$$\frac{3t^3 - 6t^2 + 4}{6}, 0 < t \leq 1$$

$$\int_{t-1}^{0} (\tau - t + 1)(\tau + 1)d\tau + \int_{0}^{t} (\tau - t + 1)(-\tau + 1)d\tau + \int_{t}^{1} (-\tau + t + 1)(-\tau + 1)d\tau$$

$$= \int_{t-1}^{0} (\tau^2 + (2-t)\tau - t + 1)d\tau + \int_{0}^{t} (-\tau^2 + t\tau - t + 1)d\tau + \int_{t}^{1} (\tau^2 - (2+t)\tau + t + 1)d\tau$$

$$= \left.\frac{\tau^3}{3}\right|_{t-1}^{0} + (2-t)\left.\frac{\tau^2}{2}\right|_{t-1}^{0} + (-t+1)\left.\tau\right|_{t-1}^{0} - \left.\frac{\tau^3}{3}\right|_{0}^{t} + t\left.\frac{\tau^2}{2}\right|_{0}^{t} + (-t+1)\left.\tau\right|_{0}^{t} + \left.\frac{\tau^3}{3}\right|_{t}^{1}$$

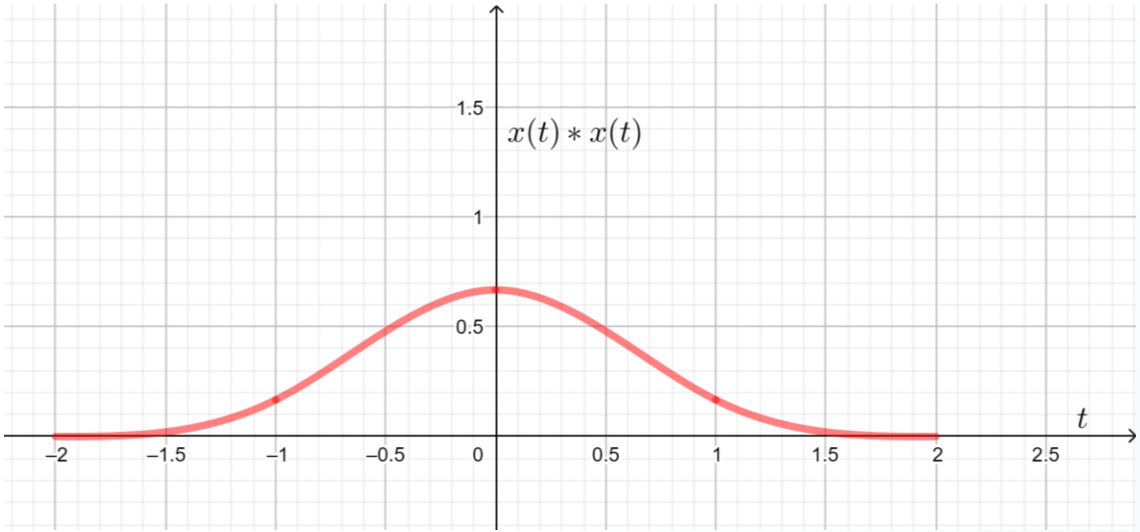
$$-(2+t)\left.\frac{\tau^2}{2}\right|_{t}^{1} + (t+1)\left.\tau\right|_{t}^{1}$$



$$-\frac{t^3-6t^2+12t-8}{6}, 1<t\leq 2$$

$$\int_{t-1}^{1}(\tau-t+1)(-\tau+1)d\tau=\int_{t-1}^{1}(-\tau^2+t\tau-t+1)d\tau=$$

$$=-\frac{\tau^3}{3}\bigg|_{t-1}^{1}+t\frac{\tau^2}{2}\bigg|_{t-1}^{1}+(-t+1)\,\tau\Big|_{t-1}^{1}$$



$$x(t) * x(t) = \begin{cases} \dfrac{t^3 + 6t^2 + 12t + 8}{6} & -2 < t \leq -1 \\ -\dfrac{3t^3 + 6t^2 - 4}{6} & -1 < \mathrm{t} \leq 0 \\ \dfrac{3t^3 - 6t^2 + 4}{6} & 0 < \mathrm{t} \leq 1 \\ -\dfrac{t^3 - 6t^2 + 12t - 8}{6} & 1 < \mathrm{t} \leq 2 \end{cases}$$

¡Muchas Gracias!