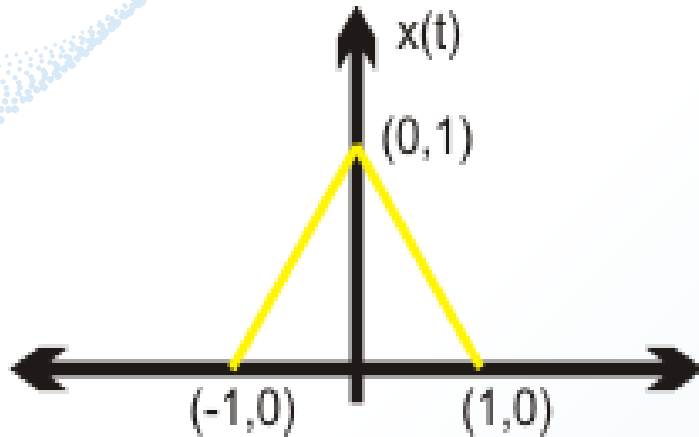


Convolución de Tiempo Continuo



Ejemplo

Utilizando las siguientes señales, se pide calcular la siguiente convolución de tiempo continuo: $x(t) * x(t)$



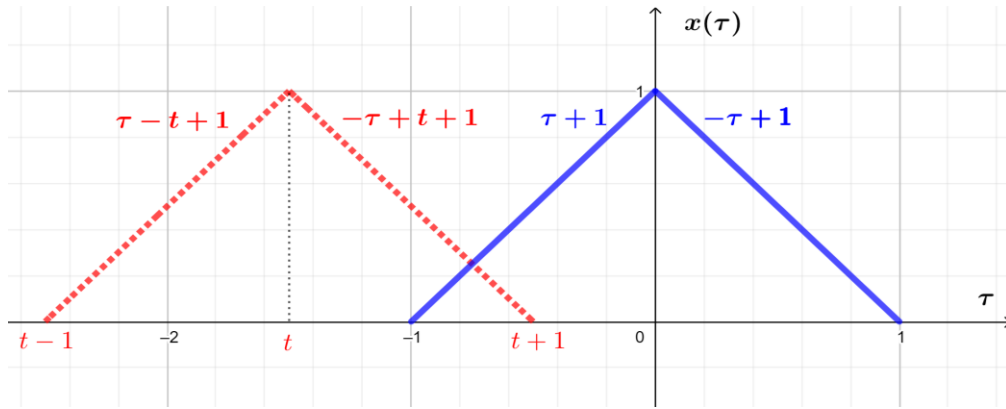
$$x(t) = \begin{cases} t + 1 & -1 < t \leq 0 \\ -t + 1 & 0 < t \leq 1 \\ 0 & \forall t \end{cases}$$

Desarrollo:

- Primeramente, debemos recordar que la misma está definida como:

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t - \tau) d\tau$$

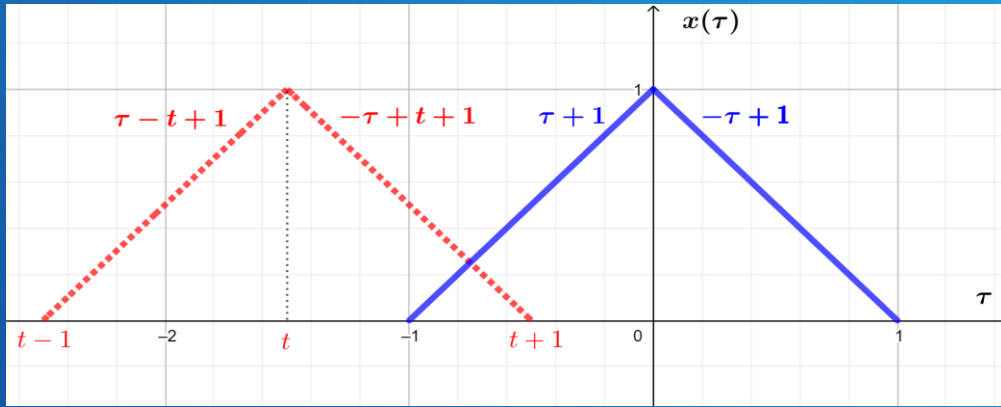
- $x(t) * x(t) = \int_{-\infty}^{\infty} x(\tau) \cdot x(t - \tau) d\tau$



Convolución Continua

$$x(\tau) = \begin{cases} \tau + 1 & -1 < \tau \leq 0 \\ -\tau + 1 & 0 < \tau \leq 1 \\ 0 & \forall \tau \end{cases}$$

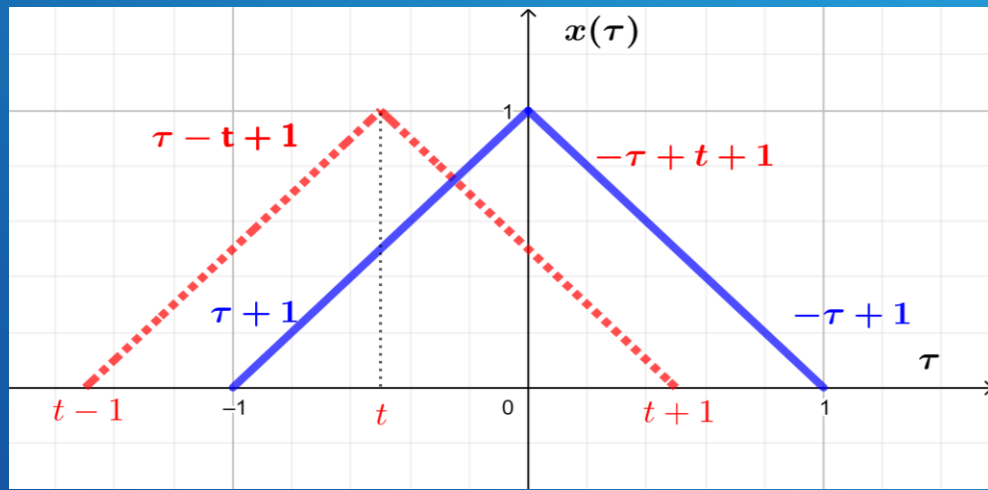
$$x(t - \tau) = \begin{cases} \tau - t + 1 & t - 1 < \tau \leq t \\ -\tau + t + 1 & t < \tau \leq t + 1 \\ 0 & \forall \tau \end{cases}$$



$$\frac{t^3 + 6t^2 + 12t + 8}{6} \quad -2 < t \leq -1$$

$$\int_{-1}^{t+1} (-\tau + t + 1)(\tau + 1) d\tau = \int_{-1}^{t+1} (-\tau^2 + t\tau + t + 1) d\tau$$

$$= -\frac{\tau^3}{3} \Big|_{-1}^{t+1} + t \frac{\tau^2}{2} \Big|_{-1}^{t+1} + (t+1) \tau \Big|_{-1}^{t+1}$$

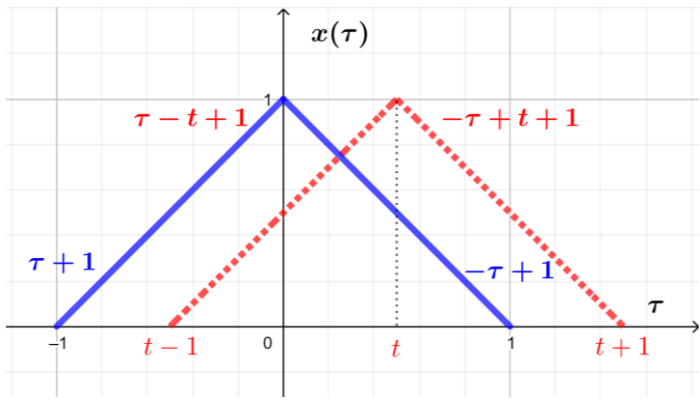


$$-\frac{3t^3 + 6t^2 - 4}{6}, -1 < t \leq 0$$

$$\int_{-1}^t (\tau - t + 1)(\tau + 1) d\tau + \int_t^0 (-\tau + t + 1)(\tau + 1) d\tau + \int_0^{t+1} (-\tau + t + 1)(-\tau + 1) d\tau$$

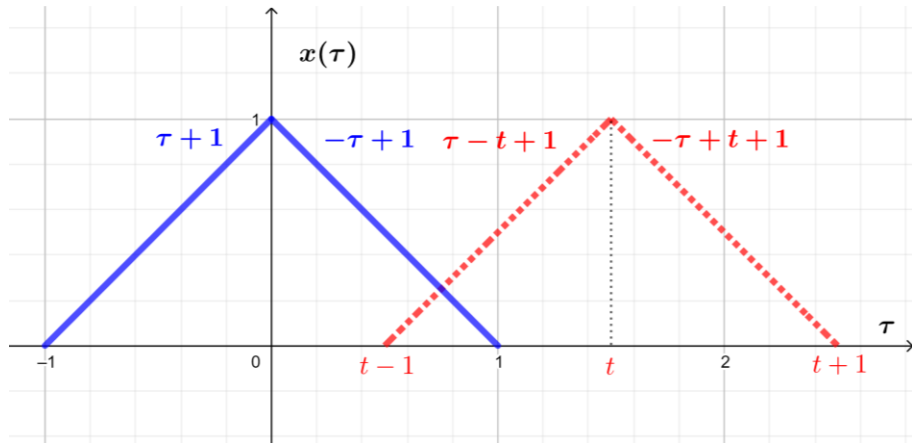
$$\int_{-1}^t (\tau^2 + (2-t)\tau - t + 1) d\tau + \int_t^0 (-\tau^2 + t\tau + t + 1) d\tau + \int_0^{t+1} (\tau^2 - (2+t)\tau + t + 1) d\tau$$

$$= -\frac{\tau^3}{3} \Big|_{-1}^t + (2-t) \frac{\tau^2}{2} \Big|_{-1}^t + (-t+1) \tau \Big|_{-1}^t - \frac{\tau^3}{3} \Big|_t^0 + t \frac{\tau^2}{2} \Big|_t^0 + (t+1) \tau \Big|_t^0 + \frac{\tau^3}{3} \Big|_0^{t+1} - (2+t) \frac{\tau^2}{2} \Big|_0^{t+1} + (t+1) \tau \Big|_0^{t+1}$$



$$\frac{3t^3 - 6t^2 + 4}{6}, 0 < t \leq 1$$

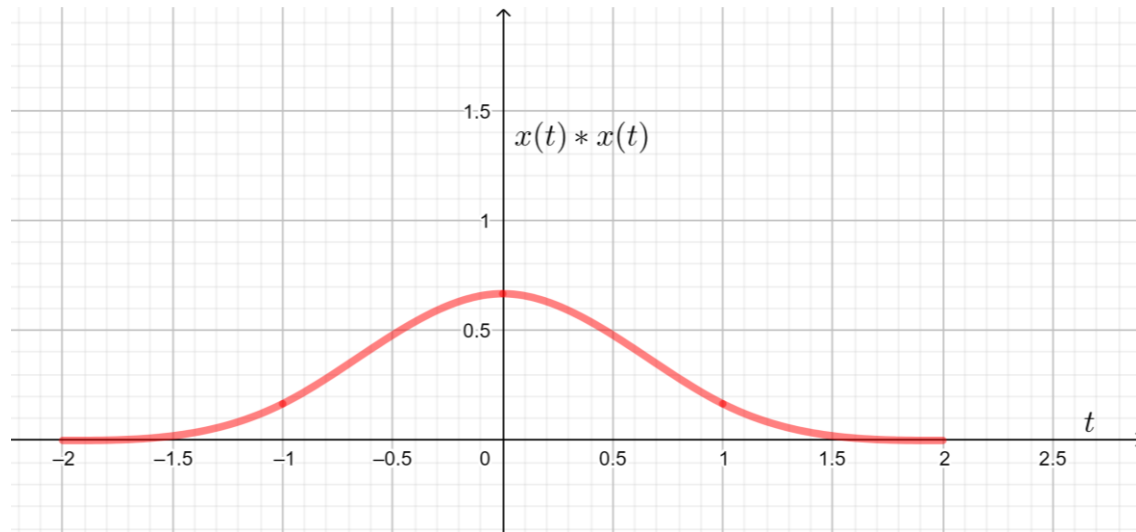
$$\begin{aligned} & \int_{t-1}^0 (\tau - t + 1)(\tau + 1) d\tau + \int_0^t (\tau - t + 1)(-\tau + 1) d\tau + \int_t^1 (-\tau + t + 1)(-\tau + 1) d\tau \\ &= \int_{t-1}^0 (\tau^2 + (2 - t)\tau - t + 1) d\tau + \int_0^t (-\tau^2 + t\tau - t + 1) d\tau + \int_t^1 (\tau^2 - (2 + t)\tau + t + 1) d\tau \\ &= \left. \frac{\tau^3}{3} \right|_{t-1}^0 + (2 - t) \left. \frac{\tau^2}{2} \right|_{t-1}^0 + (-t + 1) \tau \Big|_{t-1}^0 - \left. \frac{\tau^3}{3} \right|_0^t + t \left. \frac{\tau^2}{2} \right|_0^t + (-t + 1) \tau \Big|_0^t + \left. \frac{\tau^3}{3} \right|_t^1 \\ & \quad - (2 + t) \left. \frac{\tau^2}{2} \right|_t^1 + (t + 1) \tau \Big|_t^1 \end{aligned}$$



$$-\frac{t^3 - 6t^2 + 12t - 8}{6}, 1 < t \leq 2$$

$$\int_{t-1}^1 (\tau - t + 1)(-\tau + 1) d\tau = \int_{t-1}^1 (-\tau^2 + t\tau - t + 1) d\tau =$$

$$= -\frac{\tau^3}{3} \Big|_{t-1}^1 + t \frac{\tau^2}{2} \Big|_{t-1}^1 + (-t + 1) \tau \Big|_{t-1}^1$$



$$x(t) * x(t) = \begin{cases} \frac{t^3 + 6t^2 + 12t + 8}{6} & -2 < t \leq -1 \\ -\frac{3t^3 + 6t^2 - 4}{6} & -1 < t \leq 0 \\ \frac{3t^3 - 6t^2 + 4}{6} & 0 < t \leq 1 \\ -\frac{t^3 - 6t^2 + 12t - 8}{6} & 1 < t \leq 2 \end{cases}$$

¡Muchas Gracias!

A decorative graphic consisting of multiple parallel, wavy lines of small blue dots. The dots are arranged in a pattern that resembles a sine wave or a series of overlapping curves, creating a sense of motion and depth. The background is a solid, deep blue color.