María Florencia Zabaloy y Carina Guzowski
Facultad de Ingeniería – UnaM
Marzo 2022

APUNTE: DESARROLLO MATEMÁTICO DEL MÉTODO DE DESCOMPOSICIÓN LMDI

Curso de posgrado "El análisis de descomposición basado en índices: una herramienta para medir eficiencia energética"

Demostración caso aditivo

$$E = \sum_{i=1}^{n} E_i = \sum_{i=1}^{n} \left(P.\frac{Y}{P}.\frac{Y_i}{Y}.\frac{E_i}{Y_i} \right) = \sum_{i=1}^{n} EP.EA.ES_i.EI_i$$

$$E^T - E^0 = \Delta Etot = \Delta E_{pop} + \Delta E_{act} + \Delta E_{str} + \Delta E_{int}$$

$$E^T - E^0 = \sum_{i=1}^{n} \left(E_i^T - E_i^0 \right)$$

$$E^T - E^0 = \sum_{i=1}^{n} \frac{\left(E_i^T - E_i^0 \right)}{\left(lnE_i^T - lnE_i^0 \right)} \left(lnE_i^T - lnE_i^0 \right)$$

$$E^T - E^0 = \sum_{i=1}^{n} \frac{\left(E_i^T - E_i^0 \right)}{\left(lnE_i^T - lnE_i^0 \right)} \left[ln\left(\frac{E^T}{E^0} \right) \right]$$

$$E^T - E^0 = \sum_{i=1}^{n} \frac{\left(E_i^T - E_i^0 \right)}{\left(lnE_i^T - lnE_i^0 \right)} \left[ln\left(\frac{EP^T}{EP^0}.\frac{EA^T}{EA^0}.\frac{ES_i^T}{ES_i^0}.\frac{EI_i^T}{EI_i^0} \right) \right]$$

$$E^T - E^0 = \sum_{i=1}^{n} \frac{\left(E_i^T - E_i^0 \right)}{\left(lnE_i^T - lnE_i^0 \right)} \left[ln\left(\frac{EP^T}{EP^0} \right) + ln\left(\frac{EA^T}{EA^0} \right) + ln\left(\frac{ES_i^T}{ES_i^0} \right) + ln\left(\frac{EI_i^T}{EI_i^0} \right) \right]$$

$$w_i = \frac{\left(E_i^T - E_i^0 \right)}{\left(lnE_i^T - lnE_i^0 \right)}$$

$$E^T - E^0 = \sum_{i=1}^{n} w_i \ ln\left(\frac{EP^T}{EP^0} \right) + \sum_{i=1}^{n} w_i \, ln\left(\frac{EA^T}{EA^0} \right) + \sum_{i=1}^{n} w_i \ ln\left(\frac{ES_i^T}{ES_i^0} \right) + \sum_{i=1}^{n} w_i \ ln\left(\frac{EI_i^T}{EI_i^0} \right)$$

$$E^T - E^0 = \Delta E_{pop} + \Delta E_{act} + \Delta E_{str} + \Delta E_{int}$$

Demostración caso multiplicativo

$$E = \sum_{i=1}^{n} E_i = \sum_{i=1}^{n} \left(P.\frac{Y}{P}.\frac{Y_i}{Y}.\frac{E_i}{Y_i} \right) = \sum_{i=1}^{n} EP.EA.ES_i.EI_i$$

$$\frac{E^T}{E^0} = D_{tot} = D_{pop}.D_{act}.D_{str}.D_{int}$$

$$E^T - E^0 = \sum_{i=1}^{n} \left(E_i^T - E_i^0 \right)$$

$$E^T - E^0 = \sum_{i=1}^{n} \frac{\left(E_i^T - E_i^0 \right)}{\left(lnE_i^T - lnE_i^0 \right)} \left(lnE_i^T - lnE_i^0 \right)$$

$$E^T - E^0 = \sum_{i=1}^{n} \frac{\left(E_i^T - E_i^0 \right)}{\left(lnE_i^T - lnE_i^0 \right)} \left[ln\left(\frac{E^T}{E^0} \right) \right]$$

$$(ln\, E^T - ln\, E^0)(E^T - E^0) = \sum_{i=1}^{n} \frac{\left(E_i^T - E_i^0 \right)}{\left(lnE_i^T - lnE_i^0 \right)} \ (ln\, E^T - ln\, E^0) \left[ln\left(\frac{E^T}{E^0} \right) \right]$$

$$(ln\, E^T - ln\, E^0) = \sum_{i=1}^{n} \frac{\left(E_i^T - E_i^0 \right)}{\left(lnE_i^T - lnE_i^0 \right)} \ \frac{(ln\, E^T - ln\, E^0)}{(E^T - E^0)} \left[ln\left(\frac{E^T}{E^0} \right) \right]$$

$$ln\left(\frac{E^T}{E^0} \right) = \sum_{i=1}^{n} \frac{\frac{\left(E_i^T - E_i^0 \right)}{\left(lnE_i^T - lnE_i^0 \right)}}{\frac{(E^T - E^0)}{(ln\, E^T - ln\, E^0)}} \ \left[ln\left(\frac{EP^T}{EP^0}.\frac{EA^T}{EA^0}.\frac{ES_i^T}{ES_i^0}.\frac{EI_i^T}{EI_i^0} \right) \right]$$

$$ln\left(\frac{E^T}{E^0} \right) = \sum_{i=1}^{n} \frac{\frac{\left(E_i^T - E_i^0 \right)}{\left(lnE_i^T - lnE_i^0 \right)}}{\frac{(E^T - E^0)}{(ln\, E^T - ln\, E^0)}} \ \left[ln\left(\frac{EP^T}{EP^0} \right) + ln\left(\frac{EA^T}{EA^0} \right) + ln\left(\frac{ES_i^T}{ES_i^0} \right) + \ ln\left(\frac{EI_i^T}{EI_i^0} \right) \right]$$

$$\left(\frac{E^T}{E^0} \right) = exp\left\{ \sum_{i=1}^{n} \frac{\frac{\left(E_i^T - E_i^0 \right)}{\left(lnE_i^T - lnE_i^0 \right)}}{\frac{(E^T - E^0)}{(ln\, E^T - ln\, E^0)}} \ \left[ln\left(\frac{EP^T}{EP^0} \right) + ln\left(\frac{EA^T}{EA^0} \right) + ln\left(\frac{ES_i^T}{ES_i^0} \right) + \ ln\left(\frac{EI_i^T}{EI_i^0} \right) \right] \right\}$$

$$\widetilde{w}_i = \frac{\frac{\left(E_i^T - E_i^0\right)}{\left(lnE_i^T - lnE_i^0\right)}}{\frac{(E^T - E^0)}{(ln\,E^T - ln\,E^0)}}$$

$$\left(\frac{E^T}{E^0}\right) = exp\left\{\sum_{i=1}^{n} \widetilde{w}_i \left[ln\left(\frac{EP^T}{EP^0}\right) + ln\left(\frac{EA^T}{EA^0}\right) + ln\left(\frac{ES_i^T}{ES_i^0}\right) + ln\left(\frac{EI_i^T}{EI_i^0}\right)\right]\right\}$$

$$\left(\frac{E^T}{E^0}\right)$$

$$= exp\left[\sum_{i=1}^{n} \widetilde{w}_i\, ln\left(\frac{EP^T}{EP^0}\right)\right].exp\left[\sum_{i=1}^{n} \widetilde{w}_i\, ln\left(\frac{EA^T}{EA^0}\right)\right].exp\left[\sum_{i=1}^{n} \widetilde{w}_i\, ln\left(\frac{ES_i^T}{ES_i^0}\right)\right].exp\left[\sum_{i=1}^{n} \widetilde{w}_i\, ln\left(\frac{EI_i^T}{EI_i^0}\right)\right]$$

$$\frac{E^T}{E^0} = D_{pop}\,.D_{act}\,.D_{str}\,.D_{int}$$

Fuente: Zabaloy, M. F. (2020). Políticas Públicas de Eficiencia Energética en el Sector Residencial Argentino: el rol de las condiciones de borde y habilitantes. Bahía Blanca: Universidad Nacional del Sur. Tesis de Doctor en Economía.