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# LMDI decomposition approach: A guide for implementation

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H I G H L I G H T S

- Guidelines for implementing LMDI decomposition approach are provided.
- Eight LMDI decomposition models are summarized and compared.
- The development of the LMDI decomposition approach is presented.
- The latest developments of index decomposition analysis are briefly reviewed.

A R T I C L E I N F O

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A B S T R A C T

Since it was first used by researchers to analyze industrial electricity consumption in the early 1980s, index decomposition analysis (IDA) has been widely adopted in energy and emission studies. Lately its use as the analytical component of accounting frameworks for tracking economy-wide energy efficiency trends has attracted considerable attention and interest among policy makers. The last comprehensive literature review of IDA was reported in 2000 which is some years back. After giving an update and presenting the key trends in the last 15 years, this study focuses on the implementation issues of the logarithmic mean Divisia index (LMDI) decomposition methods in view of their dominance in IDA in recent years. Eight LMDI models are presented and their origin, decomposition formulae, and strengths and weaknesses are summarized. Guidelines on the choice among these models are provided to assist users in implementation.



## 1. Index decomposition analysis

Index decomposition analysis (IDA) was first used by researchers to study electricity consumption trends in industry in the early 1980s. The objective was to disentangle the impact on electricity consumption of changes in industrial output structure from that in industrial sector energy intensities. Since then there has been tremendous growth in the number of publications in this research area. Several literature reviews have been reported. Ang and Zhang (2000) provide a comprehensive review which covers both the methodological and application fronts. Other and more recent reviews focus on specific sub-areas. For instance, Liu and Ang (2007) deal with industrial energy analysis, while Xu and Ang (2013) concentrate on energy-related $CO_2$ emissions.

The literature review by Ang and Zhang (2000) lists 87 journal articles up to 1999 that can be appropriately classified under IDA. It is still the most comprehensive review of IDA to date. Our latest count shows that the number has increased to 559 through 2014.[1] The breakdown by time period is as follows: 55 prior to 1995, 125 from 1995 to 2004, and 379 from 2005 to 2014 (all years inclusive). The growth has been exponential, especially in the last ten years. In addition, there have been many reports with a strong policy focus released by research institutes, national agencies, and international organizations. The evidence that IDA is a useful tool in energy analysis and decision making, some 30 years after it was introduced, is strong and growing.

With the increasing maturity of the IDA methodology and changes in the global energy scene, several developments in IDA application can be observed over time. Prior to 1990, the main focus of researchers was on studying the relative impacts of changes in the aggregate level of a group of industrial activities, activity structure of the group, and activity energy intensities on energy consumption. Studies on other energy consuming sectors, namely transportation, residential, and service, started to emerge after the early 1990s. At the same time, after 1990, rising concerns about global warming have led to increased use of IDA in energy-related $CO_2$ emission studies. The growth in $CO_2$ emission studies has been very strong. Indeed since 2000 there have been more IDA journal articles dealing with emissions than energy. In the past ten years, application of IDA has also gone beyond the traditional areas of energy and emissions. New areas

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[1] In this study, journal articles cover only those written in English and appear in archival peer-reviewed academic journals. The IDA publication statistics given in various places of this paper have been generated based on the 559 titles that have been collected by the author.

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reported include water use, material and non-energy resource requirements, food production, pollutant emissions, and toxic chemical management. See, for example, Fujii and Managi (2013), Kastnera et al. (2012), Oladosu et al. (2011), Pothen and Schymura (2015), and Zhao and Chen (2014). Traditionally IDA has been used to analyze past developments, i.e. retrospective analysis of changes of an aggregate. Lately there have been a growing number of studies that deal with "prospective analysis". The three main applications are as follows. The first is making future forecasts on the basis of the decomposed effects obtained in retrospective analysis (Lescaroux, 2013; O'Mahony et al., 2013). The second is unraveling projected energy savings or reduced emissions for a future year by effect through decomposing the differences between the projected energy consumption or emission levels for the year for two different scenarios, where one of the scenarios is often the business-as-usual case (Gambhir et al., in press; Kesicki, 2013; Smit et al., 2014). The third is harmonizing and comparing projection results across different models and scenarios through quantifying the underlying drivers or effects which provide a common basis for comparisons (Föster et al., 2013; Hasanbeigi et al., 2014; Park et al., 2013).

Another important development is the use of IDA as the analytical component of the accounting framework to track economy-wide energy efficiency trends. This began in the 1990s following the initiatives undertaken by a number of national and international organizations, including the International Energy Agency (1997) and the Office of Energy Efficiency (2013) of Canada.[2] Since then, national-level studies have been undertaken in a number of other countries, including Australia, New Zealand and the United States (Ang et al., 2010). More recently, IDA was adopted by the International Energy Agency in a special focus on energy efficiency in the World Energy Outlook 2012 (International Energy Agency, 2012) and Energy Efficiency Market Report 2014 (International Energy Agency, 2014), as well as by the European Union in the Industrial Competitiveness Study 2012 (European Commission, 2012). IDA is presently being used by the World Bank and collaboration agencies as the tool for tracking progresses made in energy efficiency globally in the Global Tracking Framework of Sustainable Energy for All (SE4ALL, 2013).[3] The latest SE4ALL global tracking framework report can be found in International Energy Agency and the World Bank (2015).

The term "index decomposition analysis" was coined in Ang and Zhang (2000). It has since been widely accepted to represent what had formerly been known as "decomposition analysis" or "factorization analysis". The study points out that adding the word "index" before "decomposition analysis" is to differentiate this line of work from that of structural decomposition analysis (SDA) which is based on input–output analysis.[4] The basic principle of IDA has strong linkages with index number problems in statistics and economics. The underlying concept was largely formalized in the 1980s. Refinement and extensions to the technique have been regularly made by researchers. Examples are the search for methods that produce decomposition results without leaving a residual term, catering to cases where decomposition involves many factors or effects, spatial decomposition analysis, integrating physical and economic activity indicators in a decomposition exercise, ensuring consistency in sector aggregation when the data set has more than one level of sector aggregation, and attribution analysis of the estimated impacts by sub-sector or sub-category.

With such refinement and the need to cater to a wider range of application areas and problems, there has also been convergence with regard to IDA methods used by researchers. Prior to 1990, decomposition analysis was conducted largely based on the concept of the Laspeyres index. In the 1990s, a gradual shift towards the Divisia index was observed, or more specifically towards the method proposed by Boyd et al. (1988) which has later been referred to as the arithmetic mean Divisia index (AMDI) method. Since 2000, the most popular IDA approach has been the logarithmic mean Divisia index (LMDI) methods. The LMDI decomposition methods were adopted in two-thirds of the 254 IDA journal papers published over the five-year period from 2010 to 2014. On an annual basis, the share of papers using LMDI has been rising, from 50 percent in 2010 to 76 percent in 2014. The trend indicates that LMDI is likely to further increase its dominance over time.[5]

## 2. The LMDI decomposition approach

The LMDI decomposition approach comprises two different methods, LMDI-I and LMDI-II. The difference between them lies in the weights formulae used. In each case several decomposition models have been reported. The first model was proposed in 1997 and the term "LMDI" was introduced a year later in 1998. The two methods, LMDI-I and LMDI-II, were only formally introduced in 2001. The popularity of the LMDI approach stems from a number of desirable properties it possesses (Ang, 2004) which will be presented in later sections. A practical guide to LMDI-I is reported in Ang (2005). With LMDI now firmly established as the preferred approach in IDA, it is timely to conduct stocktaking by providing a precise and definitive documentation of the various LMDI models, including their origin, basic formulae, and key features. This will help potential users to make sensible choices and decisions when implementing it in their studies.

For both LMDI-I and LMDI-II, a decomposition analysis problem can be formulated either additively or multiplicatively. In additive decomposition analysis, the arithmetic (or difference) change of an aggregate indicator such as total energy consumption is decomposed. The aggregate change and decomposition results are given in a physical unit. In multiplicative decomposition analysis the ratio change of an aggregate indicator is decomposed. In this case, the aggregate change and decomposition results are expressed in indexes.

Furthermore, other than a quantity indicator such as energy consumption, the aggregate indicator whose change is to be decomposed can be an intensity indicator, such as energy use per value-added (for industry), per passenger-kilometer (for passenger transportation), or per unit floor space (for the residential sector).

[2] Office of Energy Efficiency (2013) is the 16th edition reporting on the national energy efficiency studies initiative undertaken by Canada that started in the 1990s.

[3] SE4ALL is a global initiative led by the Secretary-General of the United Nations to achieve universal energy access, improve energy efficiency, and increase the use of renewable energy.

[4] For a study on the similarities and differences between IDA and SDA, see Hoekstra and van den Bergh (2003).

[5] Slightly less than a third of the publications from 2010 to 2014 use a variety of other IDA methods. They include the AMDI, Laspeyres index, Fisher ideal index, Shapley/Sun, generalized Fisher index, and some other *ad hoc* methods. When decomposition analysis is for an aggregate energy intensity indicator and involves only two factors to give structure and intensity effects, greater variations in the choice of IDA methods among studies are observed. The decomposition problem is similar to separating national income and product accounts to prices and quantity effects where a large variety of index numbers can be applied. For studies that involve more than two factors, which are the norm in energy-related emission IDA studies, some of these indexes, such as the Fisher ideal index, cannot be easily applied as the formulae become fairly complex. In such cases, LMDI methods tend to dominate since their formulae take the same form irrespective of the number of factors and are therefore easy to implement (see Section 3.3).

When a quantity indicator is used, the simplest and standard IDA identity has three factors, which after a decomposition exercise lead to the well-known activity, structure, and intensity effects. With an intensity indicator, the simplest IDA identity has two factors, which lead to only the structure and intensity effects. These two types of aggregate indicators require different treatments in IDA.

In short, the LMDI approach involves variations in three different dimensions: by method (LMDI-I versus LMDI-II), by decomposition procedure (additive versus multiplication decomposition), and by aggregate indicator (quantity indicator versus intensity indicator). This leads to eight LMDI decomposition models. Users must decide which model to adopt before embarking on a decomposition study. The LMDI practical guide in Ang (2005) deals only with LMDI-I and quantity indicator for both additive and multiplicative decomposition analysis. It does not cover LMDI-II and intensity indicator.

Consider a study where changes in industrial energy consumption and aggregate energy intensity are to be decomposed. The sub-category of the aggregate is industrial sector. When an energy consumption change is to be decomposed, we begin with the following IDA identity:

$$E = \sum_i E_i = \sum_i Q\frac{Q_i}{Q}\frac{E_i}{E} = \sum_i QS_iI_i, \tag{1}$$

where $E$ is the total energy consumption in industry, $Q\ (=\sum_i Q_i)$ is the total industrial activity level, and $S_i\ (=Q_i/Q)$ and $I_i\ (=E_i/Q_i)$ are respectively the activity share and energy intensity of sector $i$. In additive and multiplicative decomposition analyses, we have, respectively,

$$\Delta E_{tot} = E^T - E^0 = \Delta E_{tot} = \Delta E_{act} + \Delta E_{str} + \Delta E_{int}, \tag{2}$$

$$D_{tot} = E^T/E^0 = D_{act}D_{str}D_{int}. \tag{3}$$

where subscripts *act*, *str*, and *int* denote the effects associated with overall activity level, activity structure, and sectoral energy intensity, respectively.

When an aggregate energy intensity change is to be decomposed, we begin with the following IDA identity:

$$V = \frac{E}{Q} = \sum_i \frac{Q_i}{Q}\frac{E_i}{Q_i} = \sum_i S_iI_i, \tag{4}$$

In additive and multiplicative decomposition analyses, we have, respectively,

$$\Delta V_{tot} = V^T - V^0 = \Delta V_{str} + \Delta V_{int}, \tag{5}$$

$$U_{tot} = V^T/V^0 = U_{str}U_{int}. \tag{6}$$

The formulae for calculating the effects in Eqs. (2), (3), (5), and (6) in the eight LMDI models are summarized in Table 1. For easy reference, they are named Model 1–Model 8. These models and formulae are not new. They have actually been reported in the literature but scatter in different sources. In Table 1, the original source where each of the models was first introduced in the literature is given.

*Model 8* is the first LMDI model introduced by researchers. Proposed by Ang and Choi (1997), it is also the first IDA method that does not leave a residual in the decomposition results. This model involves the decomposition of energy intensity change. Using the concept of a logarithmic mean function as weights for aggregation as in Ang and Choi (1997), Ang et al. (1998) proposed *Model 1* to decompose energy quantity change and introduced the term "LMDI" for the first time. The literature review by Ang and Zhang (2000) introduced *Model 5*, the additive LMDI-I method for decomposing energy intensity change.[6] No distinction was made between LMDI-I and LMDI-II then, and the multiplicative LMDI method given in Ang and Zhang (2000) is actually the same as that later known as LMDI-II for decomposing energy intensity change, i.e. Model 8 in Table 1.[7]

Ang and Liu (2001) proposed *Model 2* and pointed out that the model is perfect in decomposition and consistent in aggregation. They also introduced the terms "LMDI-I" and "LMDI-II" and differentiate the two methods formally for the first time. Ang et al. (2003) consolidated perfect decomposition IDA methods and provided a set of general formulae for LMDI-II applicable to the decomposition of both energy quantity and energy intensity indicators, which leads to *Model 3*, *Model 4*, and *Model* 7. Choi and Ang (2003) introduced the formulae for *Model 6* and Model 5. Both apply LMDI-I to decompose energy intensity changes, one multiplicatively and the other additively.

## 3. Comparisons of the eight LMDI models

From Table 1 the researcher has several choices to make when using the LMDI approach. Specifically, these choices are between using a quantity and intensity indicator, between additive and multiplicative analysis, and between LMDI-I and LMDI-II. For clarity in the discussions that follow, a simple numerical example is presented. The hypothetical data in Table 2 are taken from Ang (2004). Energy consumption in industry is analyzed and for simplicity only two industrial sectors are assumed. From Year 0 to Year *T*, total energy consumption increased from 50 to 96, giving an arithmetic change of 46 and a relative change of 1.92. The aggregate energy intensity increased from 1.0 to 1.2, giving an arithmetic change of 0.2 and a ratio change of 1.2. Application of the eight LMDI models leads to the decomposition results shown in Table 3. Since all the models are prefect in decomposition, these results do not contain a residual term. In the discussions that follow, we shall use application to energy consumption as an illustration. The conclusions remain valid when extended to other applications areas, such as energy-related $CO_2$ emissions.

### 3.1. Quantity versus intensity indicator

A quantity indicator measures the absolute level of energy consumption, while an intensity indicator has an "energy efficiency" connotation. In decomposing a change in the former, an activity effect is separately specified and its impact on energy consumption estimated, which is not the case when an intensity indicator is used. In the literature, there was equal preference for the two aggregate indicators prior to 2005. Thereafter the number of publications using quantity indicator has outnumbered that using intensity indicator by two to one. The choice between quantity indicator and intensity indicator may be independent of the choice of decomposition method. In some cases the context of the study will dictate which type of indicator should be preferred. For example, studies that deal with changes in absolute $CO_2$ emissions in a country may prefer a quantity indicator, while that focus on "energy productivity" may prefer an intensity indicator. For cases where there is no clear preference of one indicator over the other, quantity indicators may be preferred for the following reasons. First, if additive decomposition analysis is to be used, decomposing an intensity indicator is generally not a good choice. Decomposing a difference change of an intensity indicator is

[6] The formulae for Method 5 are given by Eqs. (23) and (24) in Ang and Zhang (2000).

[7] The corresponding formulae are given by Eqs. (21) and (22) in Ang and Zhang (2000).

**Table 1**
Formulae for eight LMDI decomposition models.

| | | Energy consumption decomposition: $E=\sum_{i=1}^{n} Q_i S_i I_i$ | | Energy intensity decomposition: $V=E/Q=\sum_{i=1}^{n} S_i I_i$ | |
|---|---|---|---|---|---|
| | | Additive: $\Delta E_{tot}=E^T-E^0$ | Multiplicative: $D_{tot}=E^T/E^0$ | Additive: $\Delta V_{tot}=V^T-V^0$ | Multiplicative: $U_{tot}=V^T/V^0$ |
| Method | Effect | $\Delta E_{act}+\Delta E_{str}+\Delta E_{int}$ **Model 1** | $D_{act}D_{str}D_{int}$ **Model 2** | $\Delta V_{str}+\Delta V_{int}$ **Model 5** | $U_{str}U_{int}$ **Model 6** |
| | | Source: Ang et al. (1998) | Source: Ang and Liu (2001) | Source: Ang and Zhang (2000) | Source: Choi and Ang (2003) |
| LMDI-I | Activity | $\sum_i L\left(E_i^T,E_i^0\right)\ln\left(\frac{Q^T}{Q^0}\right)$ | $\exp\left(\sum_i \frac{L\left(E_i^T,E_i^0\right)}{L\left(E^T,E^0\right)}\ln\left(\frac{Q^T}{Q^0}\right)\right)$ | N.A. | N.A. |
| | Structure | $\sum_i L\left(E_i^T,E_i^0\right)\ln\left(\frac{S_i^T}{S_i^0}\right)$ | $\exp\left(\sum_i \frac{L\left(E_i^T,E_i^0\right)}{L\left(E^T,E^0\right)}\ln\left(\frac{S_i^T}{S_i^0}\right)\right)$ | $\sum_i L\left(\frac{E_i^T}{Q^T},\frac{E_i^0}{Q^0}\right)\ln\left(\frac{S_i^T}{S_i^0}\right)$ | $\exp\left(\sum_i \frac{L\left(E_i^T/Q^T,E_i^0/Q^0\right)}{L\left(V^T,V^0\right)}\ln\left(\frac{S_i^T}{S_i^0}\right)\right)$ |
| | Intensity | $\sum_i L\left(E_i^T,E_i^0\right)\ln\left(\frac{I_i^T}{I_i^0}\right)$ | $\exp\left(\sum_i \frac{L\left(E_i^T,E_i^0\right)}{L\left(E^T,E^0\right)}\ln\left(\frac{I_i^T}{I_i^0}\right)\right)$ | $\sum_i L\left(\frac{E_i^T}{Q^T},\frac{E_i^0}{Q^0}\right)\ln\left(\frac{I_i^T}{I_i^0}\right)$ | $\exp\left(\sum_i \frac{L\left(E_i^T/Q^T,E_i^0/Q^0\right)}{L\left(V^T,V^0\right)}\ln\left(\frac{I_i^T}{I_i^0}\right)\right)$ |
| | | **Model 3** | **Model 4** | **Model 7** | **Model 8** |
| | | Source: Ang et al. (2003) | Source: Ang et al. (2003) | Source: Ang et al. (2003) | Source: Ang and Choi (1997) |
| LMDI–II | Activity | $\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)L\left(E^T,E^0\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{Q^T}{Q^0}\right)$ | $\exp\left(\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{Q^T}{Q^0}\right)\right)$ | N.A. | N.A. |
| | Structure | $\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)L\left(E^T,E^0\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{S_i^T}{S_i^0}\right)$ | $\exp\left(\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{S_i^T}{S_i^0}\right)\right)$ | $\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)L\left(V^T,V^0\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{S_i^T}{S_i^0}\right)$ | $\exp\left(\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{S_i^T}{S_i^0}\right)\right)$ |
| | Intensity | $\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)L\left(E^T,E^0\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{I_i^T}{I_i^0}\right)$ | $\exp\left(\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{I_i^T}{I_i^0}\right)\right)$ | $\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)L\left(V^T,V^0\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{I_i^T}{I_i^0}\right)$ | $\exp\left(\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{I_i^T}{I_i^0}\right)\right)$ |

Note: $L(x,y)$ is the logarithmic average of two positive numbers $x$ and $y$ given by $L(x,y)=\frac{x-y}{\ln x-\ln y}$ for $x\neq y$ $= x$ for $x=y$

somewhat hard to grasp and, because of the measurement unit, the decomposition results are cumbersome to interpret. Second, in multiplicative decomposition analysis, the decomposition results for the effects, i.e. structure and intensity effects, are the same as (for LMDI-II) or very close to (for LMDI-I) those given by the decomposition analysis using the corresponding quantity indicator (see the last two rows Table 3). Since the decomposition of a change in a quantity indicator generates more results and is relatively more informative, i.e. with the additional activity effect estimate, it may therefore be preferred. Partly because of the above reasons and partly because of the shift in application areas, the number of studies using quantity indicator has outnumbered that using intensity indicator since 2005.

### 3.2. Additive versus multiplicative decomposition analysis

Prior to 2005, the proportion of journal publications using additive decomposition analysis was 61 percent, compared to 39 percent using the multiplicative counterpart. The gap has since widened, and the respective shares were 71 percent and 29 percent for publications appearing from 2005 through 2014. From the literature one can often see that the choice between the two procedures by researchers is fairly arbitrary. In practice, a difference that one will encounter is in result presentation, since the decomposition results are given in a physical unit in the additive case while in indexes in the multiplicative case. A specific consideration is therefore about the data used, whether they are time-series data or data for selected benchmark years only. Both additive and multiplicative are applicable in the former, while additive is more convenient in the latter. A distinct advantage of the LMDI approach is that the results of additive decomposition analysis and those of multiplicative decomposition analysis are closely linked. With reference to Table 1 and for energy consumption decomposition, we have:

$$\frac{\Delta E_{tot}}{\ln D_{tot}}=\frac{\Delta E_{act}}{\ln D_{act}}=\frac{\Delta V_{str}}{\ln D_{str}}=\frac{\Delta V_{int}}{\ln D_{int}} \tag{7}$$

For energy intensity decomposition, we have:

**Table 2**
An illustrative example (arbitrary units).

| | Year 0 | | | | Year $T$ | | | |
|---|---|---|---|---|---|---|---|---|
| | $E_0$ | $Y_0$ | $S_0$ | $I_0$ | $E_T$ | $Y_T$ | $S_T$ | $I_T$ |
| Sector 1 | 30 | 10 | 0.2 | 3.0 | 80 | 40 | 0.5 | 2.0 |
| Sector 2 | 20 | 40 | 0.8 | 0.5 | 16 | 40 | 0.5 | 0.4 |
| Industry | 50 | 50 | 1.0 | 1.0 | 96 | 80 | 1.0 | 1.2 |

**Table 3**
Decomposition results based on the models in Table 1 and data in Table 2.

| | Decomposition of aggregate energy consumption | | | Decomposition of aggregate energy intensity | | |
|---|---|---|---|---|---|---|
| | | LMDI-I | LMDI-II | | LMDI-I | LMDI-II |
| Additive | | Model 1 | Model 3 | | Model 5 | Model 7 |
| | $\Delta E_{tot}$ | 46.000 | 46.000 | $\Delta V_{tot}$ | 0.2000 | 0.2000 |
| | $\Delta E_{act}$ | 32.385 | 33.143 | – | – | – |
| | $\Delta E_{str}$ | 38.285 | 37.941 | $\Delta V_{str}$ | 0.5819 | 0.5902 |
| | $\Delta E_{int}$ | −24.670 | −25.084 | $\Delta V_{int}$ | −0.3819 | −0.3902 |
| Multiplicative | | Model 2 | Model 4 | | Model 6 | Model 8 |
| | $D_{tot}$ | 1.9200 | 1.9200 | $U_{tot}$ | 1.2000 | 1.2000 |
| | $D_{act}$ | 1.5829 | 1.6000 | – | – | – |
| | $D_{str}$ | 1.7210 | 1.7126 | $U_{str}$ | 1.6997 | 1.7126 |
| | $D_{\text{int}}$ | 0.7048 | 0.7007 | $U_{\text{int}}$ | 0.7060 | 0.7007 |

$$\frac{\Delta V_{tot}}{\ln U_{tot}} = \frac{\Delta V_{str}}{\ln U_{str}} = \frac{\Delta V_{int}}{\ln U_{int}} \tag{8}$$

The above relationships hold for both LMDI-I and LMDI-II.[8] The implication is that the choice between additive and multiplicative decomposition analysis when the LMDI approach is used is inconsequential since the results given by any one of the procedures can be readily converted to those of the other. Leaving this property aside and in general, the additive decomposition analysis procedure is more suited when used in conjunction with a quantity indicator, while the multiplicative procedure is more suited when used in conjunction with an intensity indicator.

### 3.3. LMDI-I versus LMDI-II

Both LMDI-I and LMDI-II share a number of desirable properties as an IDA method. They include satisfying the factor reversal test and the time reversal test in index number problems. They are easy to formulate and apply, and are zero-value and negative-value robust. Irrespective of the number of factors in the IDA identity, the decomposition formulae take the same forms as those shown in Table 1 for each model. The linkage between the results given by additive and multiplicative decomposition analysis procedures is yet another attractive feature of both methods. From the decomposition results in Table 3 and those presented in other studies, such as Choi and Ang (2012), it is known that once a choice has been made on the aggregate indicator (quantity or intensity) and the decomposition analysis procedure (additive or multiplicative), the results given by LMDI-I and LMDI-II are actually very similar. As such, in general application, there is no strong preference for one method over the other. In the literature, LMDI-I has been far more widely adopted than LMDI-II partly because of its simpler formulae. It is also the method recommended in Ang (2004) and Ang (2005).

Yet there are some subtle differences between LMDI-I and LMDI-II that may dictate a user's choice in some specific applications. LMDI-I has two additional desirable properties: consistent in aggregation (Ang and Liu, 2001) and perfect in decomposition at the subcategory level (Ang et al., 2009).[9] These are properties that LMDI-II does not possess. If any of or both these properties are deemed important in a specific application, such as in multi-level decomposition analysis where multi-level aggregation is performed and decomposition results at sub-category level are of interest, LMDI-I will be preferred. On the other hand, in the case of LMDI-II, the weights in the formulae in Model (4) and Model (8) summed to unity (Ang and Choi, 1997), a desirable property in index construction.[10] Comparing Model 8 and Model 6, the formulae in the former look "simpler" and more intuitive, i.e. the terms used to calculate the weights formulae are given in the quantity shares, and computationally Model 8 is more attractive. These advantages of LMDI-II over LMDI-I, however, do not apply when Model 3 is compared to Model 1 or when Model 4 is compared to Model 2.

### 3.4. Guidelines for implementation

From the foregoing and taking into account theoretical foundation and ease of application, the following general guidelines on the implementation of the LMDI approach may be proposed. They are taken as general and as such may not be applicable when the user has some specific preference or considerations with good reasons. First, in additive decomposition analysis where the aggregate is a quantity indicator, Model 1 is the preferred model (and is superior to Model 3). Second, in multiplicative decomposition analysis where the aggregate is an intensity indicator, Model 8 is the preferred model (and is superior to Model 6). Third, in multiplicative decomposition analysis where the aggregate is a quantity indicator, both Model 2 and Model 4 may be adopted.

[8] The relationships can be shown analytically based on the formulae in Table 1 or the numerical example in Table 3. The linkage for multiplicative LMDI-I with proof is given in Ang (2004).

[9] These properties apply to both Model 1 and Model 2.

[10] In LMDI-I, the sum of weights in the formulae in Model (2) and Model (6) is not exactly unity, although it is generally very close to unity (Choi and Ang, 2012).

However, Model 2 has an edge over Model 4 due to its desirable properties of consistent in aggregation and perfect in decomposition at the subcategory level, simpler formulae, and linkage with Model 1. Finally, additive decomposition analysis where the aggregate is an intensity indicator is generally not recommended and as such low priority may be given to Model (5) and Model (7). In short, the top choices are Model 1 and Model 8 depending on whether the aggregate studied is a quantity or intensity indicator.[11]

## 4. Conclusion

This study provides an update to IDA and a brief review of LMDI decomposition analysis. Several key trends observed since the comprehensive literature review reported in Ang and Zhang (2000) are presented. With sustained interest among researchers and policy makers in the LMDI decomposition approach, an account is given of the origin of various LMDI decomposition models and the terminology now widely used in the research area. Eight LMDI models are presented with their basic formulae summarized. Comparisons are made among the models and guidelines for potential users are developed on model selection. Recent developments show that tracking of economy-wide energy efficiency trends is an area where the LMDI decomposition approach will increasingly be applied. Furthermore, the approach is being adopted in a growing number of non-traditional areas, including in conjunction with other modeling tools in an innovative way. While the availability of quality data is a requirement for producing good empirical work, this study serves to provide a strong foundation for the implementation of IDA and the LMDI decomposition approach.

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[11] Model 1 is used in Che and Pham (2012), International Energy Agency (2012, 2014) and Office of Energy Efficiency (2013), while Model 8 in Belzer (2014).