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The LMDI approach to decomposition analysis: a practical guide

B.W. Ang*

Department of Industrial and Systems Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, Singapore

Abstract

In a recent study, Ang (Energy Policy 32 (2004)) compared various index decomposition analysis methods and concluded that the logarithmic mean Divisia index method is the preferred method. Since the literature on the method tends to be either too technical or specific for most potential users, this paper provides a practical guide that includes the general formulation process, summary tables for easy reference and examples.

Keywords: Decomposition analysis; Index decomposition analysis; Divisia index; LMDI

1. The LMDI formulation process

Let V be an energy-related aggregate. Assume that there are n factors contributing to changes in V over time and each is associated with a quantifiable variable whereby there are n variables, $x_1, x_2, \ldots, x_n$. Let subscript i be a sub-category of the aggregate for which structural change is to be studied. At the sub-category level the relationship $V_i = x_{1,i}x_{2,i}\cdots x_{n,i}$ holds. The general index decomposition analysis (IDA) identity is given by

$$V = \sum_i V_i = \sum_i x_{1,i}x_{2,i}\cdots x_{n,i}. \quad (1)$$

The aggregate changes from $V^0 = \sum_i x^0_{1,i}x^0_{2,i}\ldots x^0_{n,i}$ in period 0 to $V^T = \sum_i x^T_{1,i}x^T_{2,i}\ldots x^T_{n,i}$ in period T. In multiplicative decomposition, we decompose the ratio:

$$D_{tot} = V^T/V^0 = D_{x_1}D_{x_2}\ldots D_{x_n}. \quad (2)$$

In additive decomposition we decompose the difference:

$$\Delta V_{tot} = V^T - V^0 = \Delta V_{x_1} + \Delta V_{x_2} + \cdots + \Delta V_{x_n}. \quad (3)$$

The subscript *tot* represents the total or overall change and the terms on the right-hand side give the effects associated with the respective factors in Eq. (1).

In the logarithmic mean Divisia index (LMDI) approach,[1] the general formulae for the effect of the kth factor on the right-hand side of Eqs. (2) and (3) are respectively:

$$\begin{aligned} D_{x_k} &= \exp\left(\sum_i \frac{L(V_i^T, V_i^0)}{L(V^T, V^0)} \ln\left(\frac{x^T_{k,i}}{x^0_{k,i}}\right)\right) \\ &= \exp\left(\sum_i \frac{(V_i^T - V_i^0)/(\ln V_i^T - \ln V_i^0)}{(V^T - V^0)/(\ln V^T - \ln V^0)} \times \ln\left(\frac{x^T_{k,i}}{x^0_{k,i}}\right)\right), \end{aligned} \quad (4)$$

$$\begin{aligned} \Delta V_{x_k} &= \sum_i L(V_i^T, V_i^0) \ln\left(\frac{x^T_{k,i}}{x^0_{k,i}}\right) \\ &= \sum_i \frac{V_i^T - V_i^0}{\ln V_i^T - \ln V_i^0} \ln\left(\frac{x^T_{k,i}}{x^0_{k,i}}\right), \end{aligned} \quad (5)$$

where $L(a,b) = (a-b)/(\ln a - \ln b)$ as defined in Ang (2004).[2] The general formulae in the formulation process are summarized in Table 6 in Appendix A.

2. Two illustrative cases

Changes in industrial energy consumption may be studied by quantifying the impacts of changes in three different factors: overall industrial activity (activity effect), activity mix (structure effect) and sectoral energy intensity (intensity effect). The sub-category of the

*Tel.: +65-687-422-03; fax: +65-677-714-34.

E-mail address: iseangbw@nus.edu.sg (B.W. Ang).

[1] The LMDI is used here to refer to the logarithmic mean Divisia method I (LMDI I). A related version, the LMDI II, has a weighting scheme slightly more complex than LMDI I (Ang et al., 2003).

[2] For more on Eqs. (4) and (5), see Ang and Liu (2001) and Ang et al. (1998), respectively.

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aggregate is industrial sector. The IDA identity in Eq. (1) is

$$E = \sum_i E_i = \sum_i Q\frac{Q_i}{Q}\frac{E_i}{E} = \sum_i QS_iI_i, \tag{6}$$

where E is the total energy consumption in the industry, $Q\,(=\sum_i Q_i)$ is the total industrial activity level, and $S_i\,(=Q_i/Q)$ and $I_i\,(=E_i/Q_i)$ are, respectively, the activity share and energy intensity of sector i. From Eqs. (2) and (3),

$$D_{tot} = E^T/E^0 = D_{act}D_{str}D_{int}, \tag{7}$$

$$\Delta E_{tot} = E^T - E^0 = \Delta E_{tot} = \Delta E_{act} + \Delta E_{str} + \Delta E_{int}. \tag{8}$$

The subscripts *act*, *str* and *int* denote the effects associated with the overall activity level, activity structure and sectoral energy intensity, respectively. The LMDI formulae can be readily worked out from Table 6 and they are summarized in Table 7 in Appendix A.

Changes in CO_2 emissions from industry may be studied by quantifying the contributions from changes in five different factors: overall industrial activity (activity effect), industry activity mix (structure effect), sectoral energy intensity (intensity effect), sectoral energy mix (energy-mix effect), and CO_2 emission factors (emission-factor effect). The sub-categories of the aggregate are industrial sector and fuel type. The IDA identity in Eq. (1) may be written as

$$C = \sum_{ij} C_{ij} = \sum_{ij} Q\frac{Q_i}{Q}\frac{E_i}{Q_i}\frac{E_{ij}}{E_i}\frac{C_{ij}}{E_{ij}} = \sum_{ij} QS_iI_iM_{ij}U_{ij}, \tag{9}$$

where C is the total CO_2 emissions and C_{ij} is the CO_2 emissions arising from fuel j in industrial sector i, E_{ij} is the consumption of fuel j in industrial sector i, where $E_i = \sum_j E_{ij}$; the fuel-mix variable is given by $M_{ij}\,(=E_{ij}/E_j)$ and the CO_2 emission factor by $U_{ij}\,(=C_{ij}/E_{ij})$. From Eqs. (2) and (3), we have

$$D_{tot} = C^T/C^0 = D_{act}D_{str}D_{int}D_{mix}D_{emf}, \tag{10}$$

$$\begin{aligned}\Delta C_{tot} &= C^T - C^0\\ &= \Delta C_{act} + \Delta C_{str} + \Delta C_{int} + \Delta C_{mix} + \Delta C_{emf}.\end{aligned} \tag{11}$$

The subscripts *act, str, int, mix* and *emf*, respectively, denote the effects associated with overall activity, activity structure, sectoral energy intensity, sectoral energy mix and emission factors. The LMDI formulae are summarized in Table 8 in Appendix A.

3. Numerical examples

We collected the 1990 (Year 0) and 2000 (Year T) energy and CO_2 emission data for industry in Canada from Nyboer (2002) and Nyboer and Laurin (2002). The database includes a total of 23 industrial sectors and 14 energy sources. The aggregate CO_2 emissions in million tonnes of CO_2 ($MTCO_2$), energy consumption in petajoules (PJ) and gross industrial output in Canadian dollars (C$) are shown in Table 1.[3] The observed changes in energy consumption and CO_2 emissions are shown in the first column of Tables 2–5. The other columns of the tables give the decomposition results obtained using the decomposition formulae in Appendix A.[4]

From Tables 2 and 3, it can be seen that Canadian industrial energy consumption increased by 16.2% or 377.8 PJ from 1990 to 2000. The LMDI decomposition

Table 1
Aggregate data for Canadian industry, 1990 and 2000

Year	C ($MTCO_2$)	E (PJ)	Q (gross output, 1986 C$ billions)
1990	114.31	2336.5	295.2
2000	135.11	2714.3	442.5

Table 2
Results of industrial energy consumption decomposition for Canada, 1990–2000: multiplicative decomposition

D_{tot}	D_{act}	D_{str}	D_{int}
1.162	1.498	0.806	0.963

Table 3
Results of industrial energy consumption decomposition for Canada, 1990–2000: additive decomposition (PJ)

ΔE_{tot}	ΔE_{act}	ΔE_{str}	ΔE_{int}
377.8	1018.6	−544.7	−96.1

Table 4
Results of industry energy-related CO_2 emission decomposition for Canada, 1990–2000: multiplicative decomposition

D_{tot}	D_{act}	D_{str}	D_{int}	D_{mix}	D_{emf}
1.182	1.493	0.814	0.951	0.980	1.044

[3] Nyboer (2002) and Nyboer and Laurin (2002) do not give CO_2 emissions arising from electricity consumption. We estimated the equivalent emission factors for electricity from the 1990 and 2000 Canadian energy balances in International Energy Agency (1993, 2002), by dividing the total emissions for fuel consumption in electricity generation by the total final electricity consumption in Canada in the respective years.

[4] Due to differences in data source, industry coverage, sector classification, industrial activity measurement and decomposition technique, the additive decomposition results obtained here are different from those in Natural Resources Canada (2002).

Table 5
Results of industry energy-related CO_2 emission decomposition for Canada, 1990–2000: additive decomposition ($MTCO_2$)

ΔC_{tot}	ΔC_{act}	ΔC_{str}	ΔC_{int}	ΔC_{mix}	ΔC_{emf}
20.80	49.84	−25.58	−6.30	−2.48	5.31

Table 6
LMDI formulae for the general case with n factors

IDA identity	$V=\sum_i V_i=\sum_i x_{1,i}x_{2,i}...x_{n,i}$	
	Multiplicative decomposition	Additive decomposition
Change scheme	$D_{tot}=V^T/V^0=D_{x_1}D_{x_2}\cdots D_{x_n}$	$\Delta V_{tot}=V^T-V^0=\Delta V_{x_1}+\Delta V_{x_2}+\cdots+\Delta V_{x_n}$
LMDI formulae	$D_{x_k}=\exp\left(\sum_i \frac{(V_i^T-V_i^0)/(\ln V_i^T-\ln V_i^0)}{(V^T-V^0)/(\ln V^T-\ln V^0)}\ln\left(\frac{x_{k,i}^T}{x_{k,i}^0}\right)\right)$	$\Delta V_{x_k}=\sum_i \frac{V_i^T-V_i^0}{\ln V_i^T-\ln V_i^0}\ln\left(\frac{x_{k,i}^T}{x_{k,i}^0}\right)$

Note: (a) Where $x_{k,i}=0$, replace all the zeros in the data set by a small positive constant, e.g. between 10^{-10} and 10^{-20}. (b) It can be shown that $\ln(D_{tot})=\ln(D_{x_1})+\ln(D_{x_2})+\cdots+\ln(D_{x_n})$. (c) The following relationship holds: $\Delta V_{tot}/\ln D_{tot}=\Delta V_{x_1}/\ln D_{x_1}=\Delta V_{x_2}/\ln D_{x_2}=\cdots=\Delta V_{x_n}/\ln D_{x_n}$.

Table 7
LMDI formulae for decomposing changes in industrial energy consumption

IDA identity	$E=\sum_i E_i=\sum_i Q\frac{Q_i}{Q}\frac{E_i}{Q_i}=\sum_i QS_iI_i$	
	Multiplicative decomposition	Additive decomposition
Change scheme	$D_{tot}=E^T/E^0=D_{act}D_{str}D_{int}$	$\Delta E_{tot}=E^T-E^0=\Delta E_{act}+\Delta E_{str}+\Delta E_{int}$
LMDI formulae	$D_{act}=\exp\left(\sum_i \frac{(E_i^T-E_i^0)/(\ln E_i^T-\ln E_i^0)}{(E^T-E^0)/(\ln E^T-\ln E^0)}\ln\left(\frac{Q^T}{Q^0}\right)\right)$	$\Delta E_{act}=\sum_i \frac{E_i^T-E_i^0}{\ln E_i^T-\ln E_i^0}\ln\left(\frac{Q^T}{Q^0}\right)$
	$D_{str}=\exp\left(\sum_i \frac{(E_i^T-E_i^0)/(\ln E_i^T-\ln E_i^0)}{(E^T-E^0)/(\ln E^T-\ln E^0)}\ln\left(\frac{S_i^T}{S_i^0}\right)\right)$	$\Delta E_{str}=\sum_i \frac{E_i^T-E_i^0}{\ln E_i^T-\ln E_i^0}\ln\left(\frac{S_i^T}{S_i^0}\right)$
	$D_{int}=\exp\left(\sum_i \frac{(E_i^T-E_i^0)/(\ln E_i^T-\ln E_i^0)}{(E^T-E^0)/(\ln E^T-\ln E^0)}\ln\left(\frac{I_i^T}{I_i^0}\right)\right)$	$\Delta E_{int}=\sum_i \frac{E_i^T-E_i^0}{\ln E_i^T-\ln E_i^0}\ln\left(\frac{I_i^T}{I_i^0}\right)$

results show that the activity effect led to an increase almost three times that margin, and the much lower growth observed was due to structural change in production and a reduction in sectoral energy intensity. Reduction in sectoral energy is often taken as a measure of improvement in energy efficiency. The impact of structural change in reducing energy consumption, arising from the shift in the composition of industry output towards less energy-intensive sectors, was estimated to be nearly six times that of improvement in energy efficiency.

From Tables 4 and 5, CO_2 emissions increased by 18.2% or 20.80 $MTCO_2$ from 1990 to 2000. The LMDI decomposition results show that the activity effect led to an increase almost two and a half times that margin but, like the case of energy consumption, actual growth in emissions was much lower because of structural change in production and, to a lesser extent, reduction in sectoral energy intensity. In addition, changes in energy mix led to a reduction but changes in emission factors led to an increase in emissions. Changes in energy mix arose from a shift towards cleaner fuels in final energy use while changes in emission factors arose from an increase in the share of fossil fuels in electricity generation. Overall, the relative contributions of the five factors show the importance of the impact of industry structure change in reducing the growth of CO_2 emissions despite a substantial increase in the overall industrial output.

4. Some LMDI application issues

An attractive feature of LMDI is the ease of formulation, which can be seen from the formulae in Appendix A. The LMDI formulae can be readily derived once the IDA identity has been specified. Commercially available spreadsheet software packages

Table 8
LMDI formulae for decomposing changes in energy-related CO_2 emissions from industry

IDA identity	$C=\sum_{ij} C_{ij}=\sum_{ij} Q\frac{Q_i}{Q}\frac{E_i}{Q_i}\frac{E_{ij}}{E_i}\frac{C_{ij}}{E_{ij}}=\sum_{ij} QS_iI_iM_{ij}U_{ij}$	
	Multiplicative decomposition	Additive decomposition
Change scheme	$D_{tot}=C^T/C^0=D_{act}D_{str}D_{int}D_{mix}D_{emf}$	$\Delta C_{tot}=C^T-C^0=\Delta C_{act}+\Delta C_{str}+\Delta C_{int}+\Delta C_{mix}+\Delta C_{emf}$
LMDI formulae	$D_{act}=\exp\left(\sum_{ij}\frac{(C_{ij}^T-C_{ij}^0)/(\ln C_{ij}^T-\ln C_{ij}^0)}{(C^T-C^0)/(\ln C^T-\ln C^0)}\ln\left(\frac{Q^T}{Q^0}\right)\right)$	$\Delta C_{act}=\sum_{ij}\frac{C_{ij}^T-C_{ij}^0}{\ln C_{ij}^T-\ln C_{ij}^0}\ln\left(\frac{Q^T}{Q^0}\right)$
	$D_{str}=\exp\left(\sum_{ij}\frac{(C_{ij}^T-C_{ij}^0)/(\ln C_{ij}^T-\ln C_{ij}^0)}{(C^T-C^0)/(\ln C^T-\ln C^0)}\ln\left(\frac{S_i^T}{S_i^0}\right)\right)$	$\Delta C_{str}=\sum_{ij}\frac{C_{ij}^T-C_{ij}^0}{\ln C_{ij}^T-\ln C_{ij}^0}\ln\left(\frac{S_i^T}{S_i^0}\right)$
	$D_{int}=\exp\left(\sum_{ij}\frac{(C_{ij}^T-C_{ij}^0)/(\ln C_{ij}^T-\ln C_{ij}^0)}{(C^T-C^0)/(\ln C^T-\ln C^0)}\ln\left(\frac{I^T}{I^0}\right)\right)$	$\Delta C_{int}=\sum_{ij}\frac{C_{ij}^T-C_{ij}^0}{\ln C_{ij}^T-\ln C_{ij}^0}\ln\left(\frac{I_i^T}{I_i^0}\right)$
	$D_{mix}=\exp\left(\sum_{ij}\frac{(C_{ij}^T-C_{ij}^0)/(\ln C_{ij}^T-\ln C_{ij}^0)}{(C^T-C^0)/(\ln C^T-\ln C^0)}\ln\left(\frac{M_{ij}^T}{M_{ij}^0}\right)\right)$	$\Delta C_{mix}=\sum_{ij}\frac{C_{ij}^T-C_{ij}^0}{\ln C_{ij}^T-\ln C_{ij}^0}\ln\left(\frac{M_{ij}^T}{M_{ij}^0}\right)$
	$D_{emf}=\exp\left(\sum_{ij}\frac{(C_{ij}^T-C_{ij}^0)/(\ln C_{ij}^T-\ln C_{ij}^0)}{(C^T-C^0)/(\ln C^T-\ln C^0)}\ln\left(\frac{U_{ij}^T}{U_{ij}^0}\right)\right)$	$\Delta C_{emf}=\sum_{ij}\frac{C_{ij}^T-C_{ij}^0}{\ln C_{ij}^T-\ln C_{ij}^0}\ln\left(\frac{U_{ij}^T}{U_{ij}^0}\right)$

can be adopted to meet the computational needs and present the results in a graphical form. As an illustration, Fig. 1 shows a radar chart for the multiplicative case while Fig. 2 shows a bar chart for the additive case using the numerical results presented in Tables 4 and 5, respectively.

From Eqs. (4) and (5), the LMDI formulae contain logarithmic terms and the variables cannot have negative values. This is a limitation of LMDI but in IDA negative values seldom occur. A more likely situation is the occurrence of zero values, i.e. $x_{k,i}=0$. In the analysis in Section 3, this occurs for sectoral energy mix and CO_2 emission factors. To overcome this problem, all the zeros in the data set may be replaced by a small positive constant, e.g. between 10^{-10} and 10^{-20}, and the computation could proceed as usual. The results converge as the small positive constant approaches zero (Ang et al., 1998).

The LMDI method has several practical advantages from the application viewpoint. First, LMDI gives perfect decomposition, i.e. the results do not contain an unexplained residual term, which simplifies the result interpretation. Second, the results given by the multiplicative LMDI possess the following additive property: $\ln(D_{tot})=\ln(D_{x_1})+\ln(D_{x_2})+\cdots+\ln(D_{x_n})$. Third, there exists a simple relationship between multiplicative and additive decomposition, i.e. $\Delta V_{tot}/\ln D_{tot}=\Delta V_{x_k}/\ln D_{x_k}$ for all k, which makes separate decomposition using the multiplicative and additive schemes unnecessary. Finally, LMDI is consistent in aggregation (Ang and Liu, 2001). Estimates of an effect at the sub-group level can be aggregated to give the corresponding effect at the group level, a property useful in multi-level aggregation studies, e.g. grouping industry activities into sub-groups, countries into regions, etc.

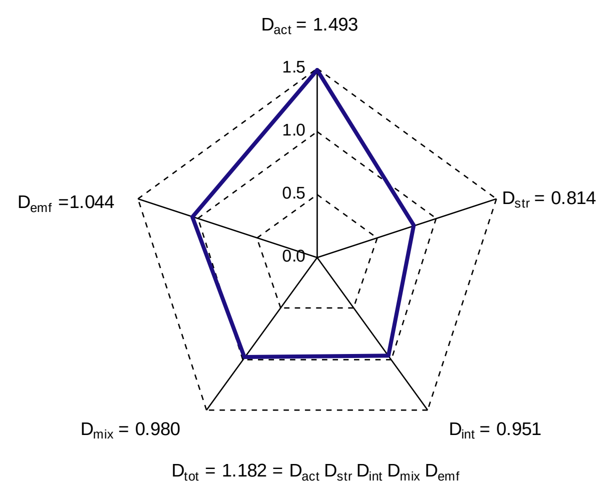

Fig. 1. Presentation of multiplication decomposition results in Table 4.

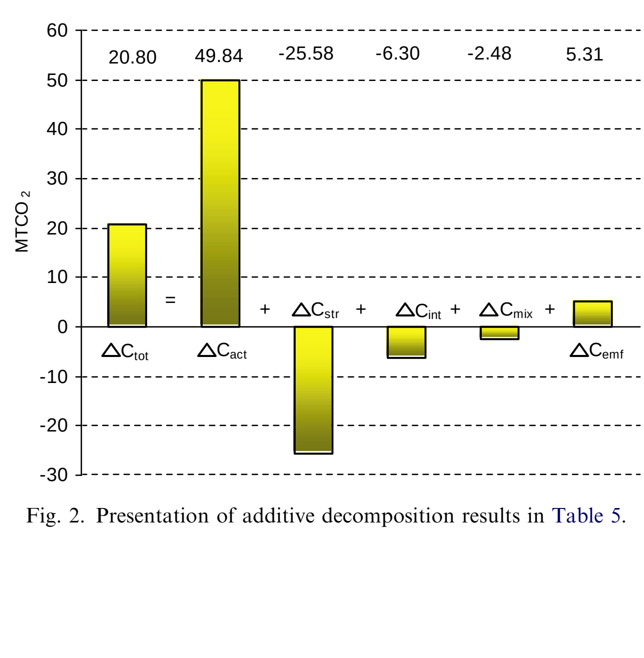

Fig. 2. Presentation of additive decomposition results in Table 5.

5. Conclusion

As a follow-up to the study by Ang (2004), this paper gives a practical guide to the LMDI decomposition approach. It will be useful to practitioners interested in adopting the approach. We summarise the general and specific LMDI formulae for ease of reference, and present two examples using real data. Some application issues are also dealt with.

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Appendix A

The LMDI formulae for the general case and the two illustrative cases in Section 2 are given in Tables 6–8.

References

Ang, B.W., 2004. Decomposition analysis for policymaking in energy: which is the preferred method? Energy Policy 32 (in press).

Ang, B.W., Liu, F.L., 2001. A new energy decomposition method: perfect in decomposition and consistent in aggregation. Energy 26, 537–548.

Ang, B.W., Zhang, F.Q., Choi, K.H., 1998. Factorizing changes in energy and environmental indicators through decomposition. Energy 23, 489–495.

Ang, B.W., Liu, F.L., Chew, E.P., 2003. Perfect decomposition techniques in energy and environmental analysis. Energy Policy 31, 1561–1566.

International Energy Agency, 1993. Energy Balances of OECD Countries 1990–1991. OECD, Paris.

International Energy Agency, 2002. Energy Balances of OECD Countries 1999–2000. OECD, Paris.

Natural Resources Canada, 2002. Energy efficiency trends in Canada 1990–2000. Office of Energy Efficiency, Ottawa, Canada.

Nyboer, J., 2002. Development of energy intensity indicators for Canadian industry 1990–2000. Canadian Industrial Energy End-use Database and Analysis Centre, Simon Fraser University, Canada.

Nyboer, J., Laurin, A., 2002. Development of greenhouse gas intensity indicators for Canadian industry supplementary 1990–2000: CO_2 emissions. Canadian Industrial Energy End-use Database and Analysis Centre, Simon Fraser University, Canada.