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Decomposition analysis for policymaking in energy: which is the preferred method?

B.W. Ang*

Department of Industrial and Systems Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, Singapore

Abstract

Although a large number of energy decomposition analysis studies have been reported in the last 25 years, there is still a lack of consensus among researchers and analysts as to which is the "best" decomposition method. As the usefulness of decomposition analysis has now been firmly established in energy studies and its scope for policymaking has expanded greatly, there is a need to have a common understanding among practitioners and consistency on the choice of decomposition methods in empirical studies. After an overview of the application and methodology development of decomposition analysis, the paper attempts to address the above-mentioned issues and provide recommendations.

Keywords: Decomposition analysis; Decomposition techniques; Divisia index; Laspeyres index

1. Introduction

Since energy researchers proposed and adopted what is now often referred to as the index decomposition analysis to study the impacts of structural change (i.e. changes in industry product mix) and sectoral energy intensity change (i.e. changes in the energy intensities of industrial sectors) on trends in energy use in industry in the late 1970s, its application has increased substantially in scope over the years. Based on the number of studies reported, index decomposition analysis is now a widely accepted analytical tool for policymaking on national energy and environmental issues. Published studies have dealt with all the OECD countries, most Eastern European countries including Russia, and a large number of developing countries ranging from Korea, China, India, Namibia, Brazil to Mexico.

An index decomposition analysis begins with defining a governing function relating the aggregate to be decomposed to a number of pre-defined factors of interest. With the governing function defined, various decomposition methods can be formulated to quantify the impacts of changes of these factors on the aggregate. After some 25 years, there is still no consensus among researchers and analysts as to which is the "best" decomposition method. In particular, there have been debates as to whether the methods based on the Divisia index are preferred to those based on the Laspeyres index, and vice versa. These are by far the two most popular decomposition approaches and in each case a number of different methods have been proposed by researchers. Not surprisingly, different methods have been adopted by international organizations, national agencies, researchers and analysts, and more often than not method selection has been made on an ad hoc basis.

Indeed, there is no simple answer to the above-mentioned question. From the theoretical foundation viewpoint, some methods can be easily shown to be superior to others. From the application viewpoint where ease of use and simplicity are important considerations, the preferred methods may be different from those preferred from the theoretical foundation viewpoint. Among the preferred methods from either the theoretical foundation viewpoint or application viewpoint, each has its strengths and weaknesses. Generally, researchers and analysts need to consider at least four issues in method selection: theoretical foundation, adaptability (e.g. the performance of a method may be data, and hence, problem specific), ease of use (e.g. whether a decomposition method can be easily applied to problems of interest), and ease of understanding and result presentation. These four issues will be explained in greater detail in Section 5.

*Tel.: +65-68742203; fax: +65-67771434.
E-mail address: iseangbw@nus.edu.sg (B.W. Ang).

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Since method selection can be problem specific, we begin by an overview of the application areas of decomposition analysis. This is followed by a discussion about the more popular decomposition methods, including those adopted by national agencies and international organizations. Finally, we present a collection of recommended methods and comment on what we believe is the most preferred method. In energy decomposition analysis, there are issues such as data quality, level of sector disaggregation, measurement of output/activity levels, and the choice of indicators which would affect the quality and validity of decomposition results. These issues are generally not method dependent and will therefore not be dealt with in our study.

2. Main application areas

Introduced in the late 1970s to study the impact of structural change on energy use in industry, index decomposition analysis has been extended and used in several other application areas for policymaking. The simplicity and flexibility of the methodology make it easy to be adopted as compared to some other decomposition methodologies, such as the input–output structural decomposition analysis where input–output tables are needed. Some 200 publications have been reported on the subject and based on these studies five main application areas may be identified, namely (a) energy demand and supply, (b) energy-related gas emissions, (c) material flows and dematerialization, (d) national energy efficiency trend monitoring, and (e) cross-country comparisons. An overview is given below with issues pertinent to method selection highlighted where appropriate. These issues will be discussed in greater details in later sections.

Of the five application areas, *energy demand and supply* which includes analysis of industrial energy demand accounts for most of the publications on decomposition analysis in the 1980s. After 1990, they also include extensions to energy demand analysis for transport, residential and in the economy, and to problems related to the energy supply sector such as the impact of fuel mix in electricity generation. Generally, basic decomposition methods would suffice for studies in this application area. The studies generally attempt to quantify the relative contributions of the impacts of structural change and energy intensity change. The definitions of these impacts may vary according to the energy sector studied. For instance, the impact of structural change concerns changes in industry product mix in the case industrial energy demand analysis. It concerns changes in transport modal mix in the case of transport energy demand analysis and changes in fuel mix in the case of electricity generation analysis.

Since 1990, an increasing number of studies on *energy-related gas emissions* decomposition have been reported. The majority of the studies dealt with energy-related carbon dioxide emissions. Methodologically, the extension from energy analysis to gas emissions analysis is rather straightforward, with gas emissions studies having more than two factors in the governing function. In addition to structural change and energy intensity change as mentioned above, they include also factors such as sectoral fuel share change and fuel gas emission coefficient change. Changes in sectoral fuel shares give the impact associated with fuel mix, whereas change in gas emission coefficients give the impact associated with fuel quality measured by carbon contents per unit of energy contents. As energy consumption is given at the individual fuel level in this application area, there are often zero values in the data set. This has implications on method selection, as some decomposition methods are unable to handle zero values.

Another extension of index decomposition analysis is the study of *material flows and dematerialization* in the national economy. Reported studies show that materials of interest include a wide range of metals and non-metallic minerals, as well as oil, coal, and natural gas which are treated as materials rather than energy sources. In these studies, energy intensity is replaced by resource use intensity given by the amount of the resource consumed per unit of economic output or value-added. Recent studies, especially in the Scandinavian countries, have found that decomposition analysis is a useful means of analyzing the development of material use in an economy.

Lately, more and more countries, such as the United States, Canada, New Zealand, and some European countries, have been developing appropriate energy efficiency indicators or indices for *national energy efficiency trend monitoring* and to measure progress towards national energy efficiency target. Index decomposition analysis has been used to single out the impact of energy intensity change using national energy consumption data involving all sectors of energy demand. Measured with reference to the level of sector disaggregation adopted, energy efficiency change may be taken as inversely proportional to energy intensity change. Recent advances in decomposition methodology, including the use of physical indicators (in addition to monetary indicators) to measure output or activity level, have helped to generate more reliable energy efficiency indicators or indices.

Cross-country comparisons involve the quantification of factors contributing to differences in energy consumption, carbon dioxide emissions, or any other aggregate between two countries or two regions. The number of studies is small but growing. The studied factors are the same as those of a single-country study, except that the data for two different years in a single-country study are now replaced by the data for two different countries for the same year. Thus for inter-country comparisons of industrial energy demand, the structure effect gives the impact arising from differences in industry product mix, and the intensity effect gives the impact arising from differences in industry sector intensities, between the two countries. There tend to be bigger variations in the data across country than over time for a single country. This leads to poor performance for some decomposition methods in cross-country comparisons.

In each of the above application areas, different decomposition methods may be used to quantify the impacts of the pre-defined factors in the governing function. It is worth-noting that the qualitative information associated with each of these factors, such as the impact of structural change or that of energy intensity change, as well as its energy, environmental or economic meaning, is the same for all the decomposition methods. However, the quantitative information, i.e. the relative contributions of the impacts measured quantitatively, is method dependent. Thus the choice of method affects the numerical results obtained despite the fact that the meanings of components are not method dependent.

3. The basic approaches

The popular decomposition methods among analysts can be divided into two groups: methods linked to the Laspeyres index and methods linked to the Divisia index. The methods used in the late 1970s and early 1980s are similar to the Laspeyres index in concept, where the impact of a factor is computed through letting that factor to change while holding all the other factors at their respective base year values. Representative examples are the studies by Jenne and Cattell (1983) and Marlay (1984) which analyzed trends in energy use in industry in the UK and US, respectively. Subsequently, extensions and refinement of methods linked to the Laspeyres index were made. Related studies include Reitler et al. (1987), Howarth et al. (1991), Park (1992), Sun (1998) and Ang et al. (2002). Boyd et al. (1987) proposed the Divisia index approach as an alternative to the Laspeyres index approach in energy decomposition analysis. Thereafter, extensions and refinement of methods linked to the Divisia index have been made. Relevant studies include Boyd et al. (1988), Liu et al. (1992), Ang (1994), Ang and Choi (1997), Ang et al. (1998), and Ang and Liu (2001).

As is well-known, the Laspeyres index measures the percentage change in some aspect of a group of items over time, using weights based on values in some base year. The Divisia index is a weighted sum of logarithmic growth rates, where the weights are the components' shares in total value, given in the form of a line integral. In simple terms, the building block of methods linked to the Laspeyres index is based on the familiar concept of percentage change whereas the building block of methods linked to the Divisia index is based on the concept of log (i.e. logarithmic) change. Törnqvist et al. (1985) presented the merit of using the log change and pointed out that it is the only symmetric and additive indicator of relative change, whereas the ordinary percentages are asymmetric and non-additive.

As an example, assume the energy consumption of an industrial sector increased from 10 units in year 0 to 20 units in year T. The relative difference calculated in the ordinary percentage depends on which of the two years is used as the point of comparison, i.e. the intensity in year T is 100% higher than in year 0, or the intensity in year 0 is 50% lower than in year T, which is asymmetric. In the case of the log change and using "ln" to denote the natural logarithm $\log_e$, the relative changes are, respectively, given by $\ln(20/10) = 0.693$ and $\ln(10/20) = -0.693$. The changes are symmetric and Törnqvist et al. (1985) recommended the use of the term "log percent" and in both cases 69.3 log percent change. The additive property of the log change will be shown in Section 6.2. In summary, the Laspeyres index is easier to understand but the Divisia index is more scientific.

4. Methods adopted by researchers and energy organizations

In the 1980s, most researchers and analysts used methods linked to the Laspeyres index. Methods linked to the Divisia index started to gain ground only in the early 1990s, and in the last 10 years reported studies using the two approaches are about equal in number. Other approaches/methods have also been proposed and applied by analysts. Some of them are mentioned in Section 5.4. Several researchers have made comparisons between different decomposition approaches or methods. They include the studies by Howarth et al. (1991), Ang and Lee (1994), Greening et al. (1997), Eichhammer and Schloman (1998), Ang and Zhang (2000), Farla and Blok (2000), Chung and Rhee (2001) and Zhang and Ang (2001). However, to date there is still a lack in uniformity and consensus in method selection, and the choice made by researchers and analysts remains rather ad hoc. In many studies, there is often no mention why a specific method has been chosen. Most authors also ignore or are probably unaware of other methods, and treat the chosen method as if it is the only one available.

In national energy efficiency trend monitoring, New Zealand has adopted a refined Divisia index method to monitor progress towards its energy efficiency target (Lermit and Jollands, 2001). The Divisia index approach has also been adopted by the US Department of Energy to construct an aggregate energy efficiency index for the United States (Wade, 2002). The European SAVE project on energy efficiency indicators has also adopted the Divisia approach to give the aggregate energy efficiency indicators for industry (ODYSSEE, 1999). Reasons given by these agencies/organizations on their choice are that the Divisia index approach is invariant to the choice of the base period and it gives only a very small residual term in the results (see Section 5).

However, the Office of Energy Efficiency (2002) has used a modified Laspeyres index approach to track and report trends in energy efficiency in Canada. The International Energy Agency (International Energy Agency, 1997; Schipper et al., 2000) has also adopted the Laspeyres index approach in their energy indicators effort. Ease of understanding is the main reason for using the Laspeyres index approach. Both the Divisia index approach and the Laspeyres index approach are referred to in the Asia Pacific Energy Research Centre (2001) project on energy efficiency indicators for the Asia Pacific Economic Cooperation (APEC) Economies.

5. Recommended methods

Fig. 1 presents a set of recommended decomposition methods linked to the Divisia index and the Laspeyres index. We have selected these methods based on the theoretical foundation and application viewpoints as explained in Section 5.1 and detailed discussions are given in Sections 5.2 and 5.3. It may be seen that decomposition can be performed multiplicatively or additively. In multiplicative decomposition the "ratio" change of an aggregate, and in the additive case its "difference" change, is decomposed. Accordingly, four categories of methods, Groups A–D, are shown in Fig. 1. Appendix A explains the difference between additive and multiplicative decomposition and gives the formulae of the respective methods in Fig. 1.

5.1. Desirable attributes of a decomposition method

As mentioned in Section 4, several researchers have compared various index decomposition methods. From their studies, the following may be taken as the criteria for evaluating the desirability of a method: (a) theoretical foundation, (b) adaptability, (c) ease of use, and (d) ease of result interpretation. Since the methods in index decomposition analysis are closely linked to index numbers, their *theoretical foundation* is based largely on that of index numbers. The following four tests in index number theory have been used by Ang et al. (2002) to determine the desirability of a decomposition method: factor-reversal, time-reversal, proportionality, and aggregation tests. Of the four tests, the most important one is the factor-reversal test and decomposition methods that pass this test have been taken by analysts as highly desirable. In addition, since decomposition can be performed additively or multiplicatively and the choice between the two is fairly arbitrary, the existence of a direct and simple association between additive and multiplicative decomposition would be viewed as a good property from the methodological viewpoint. Methods with a high degree of *adaptability* could be applied to a wide range of decomposition problems, including time-series analysis and cross-country comparisons, with little technical or

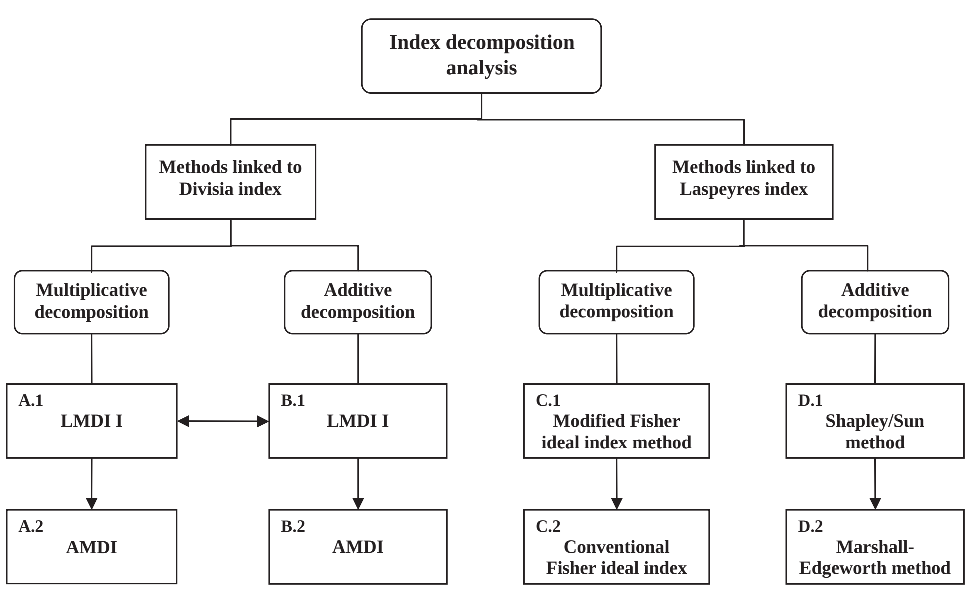

Fig. 1. Recommended methods for energy decomposition analysis.

practical difficulty. More specifically, this may be judged in terms of the values in the data set, whether a method is capable of handling date sets with large variations, zero values, or negative values. *Ease of use* concerns how easy it is for practitioners to apply a method to different problems at hand. For instance, for two different decomposition problems with different numbers of factors in the governing functions, would the formulae for the two problems given by a specific method be very similar or easy to formulate? *Ease of result interpretation* is to a great extent linked the theoretical foundation of a method and possible linkages between additive and multiplicative decomposition for the method. For instance, methods that pass the factor reversal test do not leave a residual term, which tends to complicate result interpretation. In some cases additive decomposition may be preferred to multiplicative decomposition, or vice versa, as the results can be more easily understood and communicated, and as such methods that give a direct association between additive and multiplicative decomposition could also lead to ease of result interpretation.

5.2. Methods linked to the Divisia index

In Fig. 1, methods linked to the Divisia index include Groups A and B. The log mean Divisia index method 1 (LMDI 1) is recommended for general use. Details of the multiplicative version (denoted A.1 in Fig. 1) can be found in Ang and Liu (2001) and the additive version (B.1) in Ang et al. (1998). Both versions satisfy the factor-reversal test, i.e. they give perfect decomposition whereby no unexplained residual term appears in the results. The decomposition formula takes a rather simple form, which is the same irrespective of the number of factors considered in decomposition. The linkages between the multiplicative version and the additive version can be established easily (see Section 6.2).

The arithmetic mean Divisia index methods (AMDI) use an arithmetic mean weight function where as the LMDI I use a log mean weight function. As a result, their formulae are simpler than the LMDI I counterpart. The multiplicative version (A.2) was proposed by Boyd et al. (1987) and the additive version (B.2) by Boyd et al. (1988). The AMDI may be used in place of the LMDI I in many situations and the decomposition results they give are often close to those for the LMDI I.

However, the AMDI methods have two shortcomings. First they fail the factor-reversal test. The AMDI methods can give a large residual term in the following situations: (a) cross-country decomposition where variations in the data between two countries are large, (b) yearly decomposition on a chaining basis over a long period of time where the residual term accumulates over time, and (c) decomposition on a non-chaining basis but the two decomposition years extend over a wide time span where changes in the data are significant. The second shortcoming of the AMDI methods is that they fail when the data set contains zero values, e.g. when an energy source begins or ceases to be used in a sector in the study period. The LMDI I can be shown to converge when the zero values in the data set are replaced by a small positive number but the AMDI does not have this convergence property. In any of the above-mentioned situations, the LMDI I should be used.

5.3. Methods linked to the Laspeyres index

As compared to the methods linked to the Divisia index, the linkages between multiplicative decomposition and additive decomposition in the case of the Laspeyres index approach are not as clear-cut. In multiplicative decomposition, the modified Fisher ideal index method (C.1) proposed by Ang et al. (2002) is recommended as this method gives perfect decomposition and has several desirable properties associated with the Fisher ideal index number. When decomposition involves only two factors, the modified Fisher ideal index method is identical to the Fisher ideal index number in economics (C.2). We have classified the modified Fisher ideal index method under the Laspeyres index approach for the reason that its formula has some linkages with the Laspeyres index.

In additive decomposition, the Shapley decomposition which has been used by researchers in cost allocation problems and was recently introduced by Albercht et al. (2002) to energy decomposition analysis is the recommended method (D.1). The method proposed by Sun (1998) is identical to Shapley decomposition. This method has been referred to as the refined Laspeyres index method as it involves distributing the interaction terms in the conventional Laspeyres index method to the main effects. Thus, the Shapley/Sun method has been pre-defined to give perfect decomposition. When the decomposition involves only two factors, the Shapley/Sun method is the same as the Marshall–Edgeworth method (D.2).

5.4. Methods not included in Fig. 1

We do not recommend the conventional Laspeyres index method that was used by energy researchers in the early 1980s. This method often gives a large residual, the size of which can be several times larger than the estimated effects. The adaptive weighting parametric Divisia index methods, the additive version proposed by Liu et al. (1992) and the multiplicative version by Ang (1994), are also not included in Fig. 1. These adaptive methods give a small residual term but are computationally intensive. The additive perfect decomposition method proposed by Chung and Rhee (2001) is not included in Fig. 1 because the formula is also fairly complicated. The logarithmic mean Divisia method II (LMDI II) proposed by Ang and Choi (1997) is also not included since it gives results very similar to the LMDI I counterpart, and the LMDI I is preferred for its simpler formula.

6. Illustrative example and guidelines on method selection

We compare the properties of the methods in Fig. 1 using a simple example. Although these properties can be shown analytically, the example is presented for ease of understanding. We then further discuss the issue of method selection.

6.1. An illustrative example

We use a hypothetical case where industry comprises two sectors as shown in Table 1 and the change in the aggregate energy intensity is to be decomposed to give the impacts of structural change in industrial production and sectoral energy intensity change. The notations used are as follows: E for energy consumption measured in an energy unit, Y for industrial output measured in the monetary terms, S for industrial output share, and $I = E/Y$ for energy intensity. For simplicity, these notations do not differentiate sectoral data from aggregate data. From Table 1, the sectoral energy intensity decreases for both sectors but at the industry-wide level the aggregate energy intensity increases by 20% from year 0 to year T. Sector 1, the more energy intensive of the two sectors, expands its output share from 20% in year 0 to 50% in year T.

In multiplicative decomposition, the ratio change in the aggregate energy intensity from year 0 to year T, $D_{tot} = I^{\mathrm{T}}/I^0$, is decomposed to give:

$$D_{tot} = D_{str} D_{int} D_{rsd}.$$

In additive decomposition, the difference change $\Delta I_{tot} = I^T - I^0$ is decomposed to give:

$$\Delta I_{tot} = \Delta I_{str} + \Delta I_{int} + \Delta I_{rsd}.$$

In the above, D_{str} and ΔI_{str} give the estimated impacts associated with structural change, D_{int} and ΔI_{int} give the estimated impacts associated with sectoral energy intensity change, and D_{rsd} and ΔI_{rsd} are the residual terms, respectively, for multiplicative decomposition and additive decomposition. A method gives perfect decomposition if it can be shown analytically that $D_{rsd} = 1$ (for multiplicative decomposition) and $\Delta I_{rsd} = 0$ (for additive decomposition), i.e. the method satisfies the factor-reversal test.

The decomposition results obtained using the methods shown in Fig. 1 are summarized in Tables 2 and 3. The results for the conventional Laspeyres index method (not included in Fig. 1) are also included for comparisons. Table 2 applies to the case where decomposition is performed from year 0 to year T. It can be seen that the conventional Laspeyres index method gives a large residual term; in the case of additive decomposition the residual is about of the same size as the estimated impact for sectoral energy intensity change. On the other hand the AMDI methods give relatively small residual terms.

Table 1
An illustrative example (arbitrary units)

	Year 0				Year T			
	E_0	Y_0	S_0	I_0	E_T	Y_T	S_T	I_T
Sector 1	30	10	0.2	3.0	80	40	0.5	2.0
Sector 2	20	40	0.8	0.5	16	40	0.5	0.4
Industry	50	50	1.0	1.0	96	80	1.0	1.2

Table 2
Results of decomposition from year 0 to year T using the data in Table 1

	Methods linked to Laspeyres index		Methods linked to Divisia index	
Multiplicative	Laspeyres index	Fisher ideal index (C.1)	AMDI (A.2)	LMDI I (A.1)
D_{tot}	1.2000	1.2000	1.2000	1.2000
D_{str}	1.7500	1.7078	1.6879	1.6996
D_{int}	0.7200	0.7026	0.7020	0.7060
D_{rsd}	0.9524	1[a]	1.0127	1[a]
Additive	Laspeyres index	Shapley/Sun method (D.1)	AMDI (B.2)	LMDI I (B.1)
ΔI_{tot}	0.2000	0.2000	0.2000	0.2000
ΔI_{str}	0.7500	0.6150	0.5920	0.5819
ΔI_{int}	−0.2800	−0.4150	−0.3913	−0.3819
ΔI_{rsd}	−0.2700	0[a]	−0.0007	0[a]

[a] Perfect decomposition.

Table 3
Results of decomposition from year T to year 0 using the data in Table 1

	Methods linked to Laspeyres index		Methods linked to Divisia index	
Multiplicative	Laspeyres index	Fisher ideal index (C.1)	AMDI (A.2)	LMDI I (A.1)
D_{tot}	0.8333	0.8333	0.8333	0.8333
D_{str}	0.6000	0.5855	0.5924	0.5883
D_{int}	1.4583	1.4232	1.4245	1.4164
D_{rsd}	0.9524	1[a]	0.9875	1[a]
Additive	Laspeyres index	Shapley/Sun Method (D.1)	AMDI (B.2)	LMDI I (B.1)
ΔI_{tot}	−0.2000	−0.2000	−0.2000	−0.2000
ΔI_{str}	−0.48	−0.6150	−0.5920	−0.5819
ΔI_{int}	0.55	0.4150	0.3913	0.3819
ΔI_{rsd}	−0.27	0[a]	0.0007	0[a]

[a] Perfect decomposition.

In this example, they are good substitutes for the LMDI I methods. In multiplicative decomposition the results for the Fisher ideal index method and the LMDI I are very similar, but in additive decomposition they are some differences between the results given by the Shapley/Sun method and those by the LMDI I.

Table 3 gives the case where decomposition is performed from year T to year 0. From the results in Tables 2 and 3, it can be seen that all the methods except the conventional Laspeyres index method satisfy the time-reversal test in index number theory. Satisfying the time-reversal test requires that for each estimated effect, the estimated value from year 0 to year T is the reciprocal of the estimated value from year T to year 0 in the multiplicative case, and the two estimated values are the same in absolute terms but differ only in sign in the additive case. In summary, the conventional Laspeyres methods fail both the factor-reversal test and the time-reversal test. All the methods shown in Fig. 1 pass these two tests except the AMDI methods, which pass only the time-reversal test.

6.2. Method selection

From the theoretical foundation viewpoint, the LMDI I methods are the most elegant. First, they pass the factor-reversal test and the time-reversal test. Second, the multiplicative LMDI I also possesses the additive property in the log form, i.e. $\ln(D_{tot}) = \ln(D_{str}) + \ln(D_{int})$, which can be shown from the results in Tables 2 or 3. Third, the results of the multiplicative and additive versions are linked by the following very simple and useful relationship (see Appendix A for the details):

$$\frac{\Delta V_{tot}}{\ln D_{tot}} = \frac{\Delta V_{str}}{\ln D_{str}} = \frac{\Delta V_{int}}{\ln D_{int}}.$$

With this simple relationship, once we have the estimated effect for a factor given in multiplicative decomposition, the corresponding estimated effect in additive decomposition can be readily derived, and vice versa. Theoretically, it only makes sense that there should be a simple relationship between the results given by multiplicative decomposition and by additive decomposition, so that the choice made by the analyst between the two is inconsequential.

Unlike the LMDI I, the linkages between the multiplicative modified Fisher ideal index method and the additive Shapley/Sun method are not straightforward, and hence there is the lack of a “systems” framework in formulation for methods linked to the Laspeyres index. In addition, a major difference between the LMDI I (including the AMDI) and the methods linked to the Laspeyres index approach is ease of formulation. In the case of the LMDI I, the formulae of a multi-factor problem with any number of factors take exactly the same form as those for the two-factor problem (see Appendix A). However, for the modified Fisher ideal index method and the Shapley/Sun method, the formulae have more terms as the number of factors increases. Their formulae are fairly complex when the number of factors exceeds three, and decomposition analysis is now widely applied to problems that have more than three factors. Studies on energy-related gas emissions, for instance, generally involve 4 or 5 factors.

For the reasons given in Section 6.1, we do not recommend the use of the conventional Laspeyres index method, despite the fact that it has the advantage of ease of understanding. To those who favor methods linked to the Laspeyres index approach, we would recommend the modified Fisher ideal index method and the Shapley/Sun method instead. When such a step is taken, the advantage of ease of understanding for the Laspeyres index is lost. The decomposition schemes adopted in these Laspeyres index related methods are as difficult, if not more difficult, to explain as compared to the LMDI I. For instance, it would be difficult to explain to the ordinary users how the conventional two-factor Fisher index is extended to three factors or more, and to explain the meaning of the interaction terms and the basis of distributing these terms to the main effects in the case of the Shapley/Sun method.

We mentioned the need to consider four different issues in method selection in Section 5.1. The above discussions have touched on the theoretical foundation issue, and some aspects of the other three issues. Based on the above discussions, we would consider the LMDI I the most preferred methods and would recommend them to practitioners for general use. However, in some specific applications, because of problem nature, some other methods in Fig. 1 may be used in place of the LMDI I. For instance, if the problems associated with the AMDI mentioned in Section 5.2 do not exist, the AMDI can be adopted for simplicity sake. When the data set contains negative values, which is unlikely but not impossible, it is necessary to use the methods linked to the Laspeyres index.

7. Conclusion

Decomposition analysis is a subject area that has gained in importance in policymaking in the energy field in the last 25 years. We describe its application areas, the commonly used decomposition methods, and the methods adopted by some national agencies and international organisations. As many different methods have been proposed, we present a summary of the recommended ones in a simple framework based on the Divisia index and the Laspeyres index. We discuss the properties of these methods and conclude by recommending the multiplicative and additive LMDI I methods due to their theoretical foundation, adaptability, ease of use and result interpretation, and some other desirable properties in the context of decomposition analysis. We also point out that in some specific situations, other methods may be adopted in place of the LMDI I methods.

Appendix A. A summary of decomposition formulae

Assume that V is an aggregate, there are n factors, $V=\sum_i x_{1,i}x_{2,i}\dots x_{n,i}$ and $V_i=x_{1,i}x_{2,i}\dots x_{n,i}$, where subscript i denotes an attribute of the aggregate such as energy consuming sector, fuel type, etc. Further assume that from period 0 to period T the aggregate changes from $V^0=\sum_i x^0_{1,i}x^0_{2,i}\dots x^0_{n,i}$ to $V^T=\sum_i x^T_{1,i}x^T_{2,i}\dots x^T_{n,i}$. We then have

Multiplicative decomposition:

$$D_{tot}=V^T/V^0=D_{x_1}D_{x_2}\dots D_{x_n}D_{rsd},$$

Additive decomposition:

$$\Delta V_{tot}=V^T-V^0 = \Delta V_{x_1}+\Delta V_{x_2}+\cdots+\Delta V_{x_n}+\Delta V_{rsd},$$

where D_{rsd} and ΔV_{rsd} are residual terms which may be excluded for methods that give perfect decomposition. The relevant formulae for the methods in Fig. 1 are summarized below.

A.1. Log mean Divisia index methods (LMDI I) (A.1 and B.1)

The formulae of the effect of the kth factor are:

LMDI I (multiplicative):

$$\mathrm{D}_{x_k}=\exp\left(\sum_i \frac{L(V_i^T,V_i^0)}{L(V^T,V^0)}\ln\left(\frac{x^T_{k,i}}{x^0_{k,i}}\right)\right),$$

LMDI I (additive):

$$\Delta V_{x_k}=\sum_i L\left(V_i^T,V_i^0\right)\ln\left(\frac{x^T_{k,i}}{x^0_{k,i}}\right),$$

where function $L(a,b)$ is the logarithmic average of two positive numbers a and b given by

$$L(a,b)=\frac{a-b}{\ln a-\ln b}\quad \text{for } a\neq b, \qquad =\mathrm{a}\quad \text{for } a=b.$$

Both methods give perfect decomposition. For the details, refer to Ang and Liu (2001) and Ang et al. (1998). The following simple relationship exists between multiplicative and additive decomposition:

$$\frac{\Delta V_{x_k}}{\ln D_{x_k}}=L(V^T,V^0)=\frac{V^T-V^0}{\ln(V^T/V^0)}=\frac{\Delta V_{tot}}{\ln D_{tot}}.$$

A.2. Arithmetic mean Divisia index methods (AMDI) (A.2 and B.2)

The formulae for the effect of the kth factor are:

AMDI (multiplicative):

$$D_{x_k}=\exp\left(\sum_i W_i^*\ln\left(\frac{x^T_{k,i}}{x^0_{k,i}}\right)\right).$$

AMDI (additive):

$$\Delta V_{x_k}=\sum_i W_i'\ln\left(\frac{x^T_{k,i}}{x^0_{k,i}}\right),$$

where $W_i^*=\left(V_i^T/V^T+V_i^0/V^0\right)/2$ and $W_i'=\left(V_i^T+V_i^0\right)/2$. None of the two methods gives perfect decomposition. For the details, refer to Boyd et al. (1987) and Boyd et al. (1988).

A.3. Fisher ideal index methods (C.1 and C.2)

The general formula for the modified Fisher ideal index method (C.1) is rather complicated. Interested readers can refer to Ang et al. (2002) for the details. In the two-factor case, i.e. the conventional Fisher ideal index (C.2), the formulae for the effects are:

$$D_{X_1}=\left(\frac{\sum_i x^T_{1,i}x^0_{2,i}\sum_i x^T_{1,i}x^T_{2,i}}{\sum_i x^0_{1,i}x^0_{2,i}\sum_i x^0_{1,i}x^T_{2,i}}\right)^{1/2},$$

$$D_{X_2}=\left(\frac{\sum_i x^0_{1,i}x^T_{2,i}\sum_i x^T_{1,i}x^T_{2,i}}{\sum_i x^0_{1,i}x^0_{2,i}\sum_i x^T_{1,i}x^0_{2,i}}\right)^{1/2}.$$

The methods give perfect decomposition.

A.4. Shapley/Sun method (D.1) and Marshall–Edgeworth method (D.2)

The general formula for the Shapley/Sun method (D.1) is rather complicated. Interested readers can refer to Albrecht et al. (2002), Ang et al. (2003) and Sun (1998) for the details. In the two-factor case, i.e. the Marshall–Edgeworth method (D.2), the formulae for the effects are:

$$\Delta V_{x_1}=\frac{1}{2}\left[\left(\sum_i x^T_{1,i}x^0_{2,i}-\sum_i x^0_{1,i}x^0_{2,i}\right)+\left(\sum_i x^T_{1,i}x^T_{2,i}-\sum_i x^0_{1,i}x^T_{2,i}\right)\right],$$

$$\Delta V_{x_2}=\frac{1}{2}\left[\left(\sum_i x^0_{1,i}x^T_{2,i}-\sum_i x^0_{1,i}x^0_{2,i}\right)+\left(\sum_i x^T_{1,i}x^T_{2,i}-\sum_i x^T_{1,i}x^0_{2,i}\right)\right].$$

The methods give perfect decomposition.

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